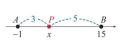# 絕對值

The Absolute Value

Materials	Notes
分點公式 設 $A(a)$, $B(b)$ 為數線上兩點。若 $P(x)$ 點在 $\overline{AB}$ 上，且 $\overline{AP}:\overline{BP}=m:n$，其中 m, n 為正數，則 P 點的坐標 $x=\dfrac{na+mb}{m+n}$。	**Vocabulary:** Section Formula (分點公式), Internal Section Formula (內分點公式), External Section Formula (外分點公式), Line Segment/ Segment (線段). **Translations:** 1. Set two points A and B located at "a" and "b" on the number line. (設 A(a), B(b)為數線上兩點。) 2. If P is located at "x" on line segment AB, then the ratio of segment AP to segment BP equals "m" to "n", where m and n are positive numbers. (若 P(x) 點在 $\overline{AB}$ 上，且 $\overline{AP}:\overline{BP}$ =m:n，其中 m, n 為正數。) 3. Therefore, point P can be represented as x equals "na" plus "mb" all over "m" plus "n". (則 P 的坐標為 x $=\dfrac{na+mb}{m+n}$ 。)
例題 1 設數線上兩點 $A(-1)$, $B(15)$。 (1)已知點 $P(x)$ 在 $\overline{AB}$ 上，且 $\overline{AP}:\overline{BP}=3:5$，求 x 的值。 (2)已知 $\overline{AB}$ 外一點 $Q(y)$ 滿足 $\overline{AQ}:\overline{BQ}=3:5$，求 y 的值。 解 (1)為 $P(x)$ 在 $\overline{AB}$ 上，利用分點公式，得 $x=\dfrac{3\times 15+5\times(-1)}{3+5}=\dfrac{40}{8}=5$。 (2)因為 Q 點在 $\overline{AB}$ 外，又 $\overline{AQ}<\overline{BQ}$，所以 A 介於 Q 和 B 之間。依題意作圖如下，得 $\overline{QA}:\overline{BA}=3:2$。點 $A(-1)$ 在 $\overline{QB}$ 上，利用分點公式，得 $-1=\dfrac{3\times 15+2\times y}{3+2}$，解得 $y=-25$。	**Translations:** 1. Set two points A and B to be located at negative 1 and 15 on a number line respectively. (設數線上兩點 A(−1), B(15)。) 2. Let P be a point located at "x" and it divides the segment AB in the ratio 3 to 5, which means the ratio of segment AP to segment BP equals 3 to 5. Find the value of x. (已知點 P(x)在 $\overline{AB}$ 上，且 $\overline{AP}:\overline{BP}$ =3:5，求 x 的值。) 3. Let Q located at "y" that divides segment AB externally in the ratio 3 to 5, which means the

ratio of segment AQ to segment BQ equals 3 to 5 . Find the value of y. (已知 $\overline{AB}$ 外一點 Q(y)滿足 $\overline{AQ}:\overline{BQ}=3:5$，求 y 的值。)

Solutions:

1. Point P at "x" is on line segment AB. By **the Section Formula**, we have x equals the quantity of 3 times 15 plus 5 times negative 1 over 3 plus 5, which is equal to 40 over 8. So the answer will be 5. (點 P(x)在 $\overline{AB}$ 上，利用分點公式得

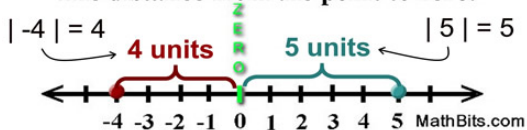 $$x = \frac{3 \times 15 + 5 \times (-1)}{3+5} = \frac{40}{8} = 5 \text{。})$$

2. Point P lies on the external part of line segment AB, and the length of segment AQ is less than segment BQ. Hence, point A lies between point Q and point B. We will have the ratio of segment QA to segment BA equals 3 to 2. (因為 Q 點在 $\overline{AB}$ 外，又 $\overline{AQ} < \overline{BQ}$，所以 A 介於 Q 和 B 之間。依題意得 $\overline{QA}:\overline{BA}=3:2$)

3. Point A located at -1 lies on segment QB. By **the Section Formula**, we will get negative 1 is equal to the quantity of 3 times 15 plus 2 times y over 3 plus 2. It will lead to the answer that y is equal to negative 25. (點 A(-1)在 $\overline{QB}$ 上，利用分點公式得

   $$-1 = \frac{3 \times 15 + 2 \times y}{3+2}\text{，解得 } y = -25 \text{。})$$

Absolute Value
The distance from the point to zero.
| -4 | = 4   4 units   5 units | 5 | = 5
-4 -3 -2 -1 0 1 2 3 4 5 MathBits.com
Distance is always positive, or zero.

Vocabulary: Unit (單位).

Sentences:

1. The absolute value of x can be represented as the distance from the point x to zero. (x 絕對值的幾何意義可以表示成 x 點到 0 的距離。)

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2. The distance is always positive, or zero. (距離一定是正數或是 0。)

3. The absolute value of negative 4 can be expressed as the distance from negative 4 to zero, which is 4 units of length. (負 4 絕對值的幾何意義可以表示成負 4 到 0 的距離，也就是 4 單位長。)

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絕對值的幾何意義與代數意義

設數線上兩點 $P(x)$ 與 $A(a)$。

(1)符號$|x|$表示數線上點 $P(x)$ 與原點 $O(0)$ 的距離，且

$$|x| = \begin{cases} x \text{，當 } x \geq 0 \\ -x \text{，當 } x < 0 \end{cases}$$

(2)符號$|x-a|$表示數線上點 $P(x)$ 與點 $A(a)$ 的距離，且

$$|x-a| = \begin{cases} x-a \text{，當 } x \geq a \\ -(x-a) \text{，當 } x < a \end{cases}$$

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Vocabulary: Absolute Value (絕對值), Abs (絕對值的縮寫), Geometry (幾何), Algebra (代數), Origin/Zero (原點).

Translations:

1. The geometric and algebraic meaning of the absolute value. (絕對值的幾何意義與代數意義。)

2. Set two points P and A to be at "x" and "a" on the number line respectively. (設數線上兩點 P(x)與 A(a)。)

3. The symbol "|x|" represents the distance from point P located at "x" to the origin on the number line. We will have abs x equals positive x when x is greater than or equal to 0. Otherwise, it equals negative x when x is less than 0. (符號|x|表示數線上點 P(x)與原點 O(0)的距離，且|x|等於 x 當 x 大於等於 0；|x|等於−x 當 x 小於 0。)

4. The symbol "|x−a|" represents the distance from point P located at "x" to point A located at "a" on the number line. We will have the abs x minus a equals x minus a when x is greater than or equal to a. Otherwise it equals the opposite of x minus a when x is less than a. (符號|x−a|表示數線上點 P(x)與原點 A(a)的距離，且|x−a |等於 x−a 當 x

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大於等於 a；|x−a| 等於 −(x−a) 當 x 小於 a。)

Notes:

1. To show that we want the absolute value of something, we put "|" marks on either side, which are called bars.

2. Sometimes absolute value is also written as "abs( )", so abs(−1) = 1 is the same as |−1| = 1.

3. The abbreviation ABS means Absolute Value.

例題 4

解方程式 $|x|+|x+3|=5$。

Vocabulary: Expression (算式), Scenario (情形).

Translations:

1. Solve the equation: The absolute value of x plus the absolute value of x plus 3 equals 5. (Or you can say: This expression plus this expression is equal to 5.) (解方程式 x 絕對值加 x 加 3 的絕對值等於 5。也可以說這個加這個等於 5。)

2. To find the changing point, we can let the expression inside of the absolute value which is x be zero, and we get x is equal to zero. (找到絕對值裡會變化的點，我們可以設絕對值裡的式子為 0，得到 x 等於 0。)

3. Zero and negative 3 divide the number line into three parts, so we have three scenarios. The first scenario is "x is greater than or equal to 0." The second scenario is "x is greater than or equal to negative 3 and less than 0". The third scenario is "x is less than 0."(0 跟 3 將數線分成 3 段討論，第一段是 x 大於等於 0，第二段是 x 介在 0 到 3 中間，第三段是 x 小於 0。)

4. In the second scenario where x is greater than or equal to negative 3 and less than 0, this makes
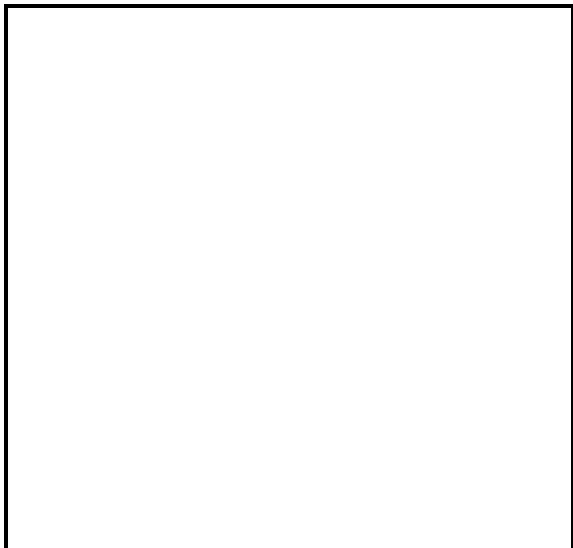
the value of the first absolute value positive. We just replace it with the expression inside itself. The expression inside the 2nd absolute value has a negative value. So we need to replace the absolute value with the opposite of the inside expression. (在第二個討論中，因為 x 介於 0 到 3 之間，所以第一個絕對值內的算式為正的，我們可以直接去掉絕對值；第二個絕對值內的算式為負的，我們去掉絕對值後將它加負號。)

絕對值不等式的解

設 k 是正數。

(1) 若 $|x| \leq k$，則 $-k \leq x \leq k$，並記作 $[-k, k]$。

(2) 若 $|x| < k$，則 $-k < x < k$，並記作 $(-k, k)$。

(3) 若 $|x| \geq k$，則 $x \geq k$ 或 $x \leq -k$，並記作 $(-\infty, -k] \cup [k, \infty)$。

(4) 若 $|x| > k$，則 $x > k$ 或 $x < -k$，並記作 $(-\infty, -k) \cup (k, \infty)$。

Vocabulary: Inequality (不等式), Interval (區間), Infinity (無限).

Translations:

1. Absolute value of inequality. (絕對值不等式)

2. If the absolute value of x is less than or equal to k, then x is between negative and positive k, denoted by brackets with the negative k and the positive k within bracket. (若 $|x| \leq k$，則 $-k \leq x \leq k$，並記作 $[-k, k]$。)

3. That also means the distance from points, which lies in the interval negative k to positive k, to 0 is less than k. (也可以看成 x 到 0 的距離在 $-k$ 到 k 的區間內。)

4. $(-\infty, b]$: From negative infinity up to and including b.

5. (a, b) : Between "a" and "b" with neither included.

6. $[a, b]$: Between "a" and "b" with both included.

7. $[a, \infty)$: From "a" up to but not including infinity.

8. $A \cup B$: A union B. / A or B.

	9.　A∩B : A intersection B. / A and B.
	Vocabulary: Endpoints (端點). **Translations:** 1. Geometric and algebraic meaning of solutions. (解的幾何及代數意義。) 2. $(-\infty, 1)\cup(5,\infty)$: The interval of negative infinity and one without including the endpoints. It's going to unite the interval of five and infinity without including the endpoints. (負無限大到 1 不包含端點的區間，聯集 5 到無限大不包含端點的區間。) 3. The distance from x to 3 is greater than 2. By the figure below we will have that subtracting 2 away from 3 is negative 1; adding 2 to 3 is five. (3 與 x 的距離大於 2。)

Supplementary Materials

Solve Applications with Absolute Value

　　Absolute value inequalities are often used in the manufacturing process. An item must be made with near perfect specifications. Usually there is a certain tolerance of the difference from the specifications that is allowed. If the difference from the specifications exceeds the tolerance, the item is rejected.

$$|actual\text{-}ideal| \le tolerance$$

Problem

　　The ideal diameter of a rod needed for a machine is 60 mm. The actual diameter can vary from the ideal diameter by 0.075 mm. What range of diameters will be acceptable to the customer without causing the rod to be rejected?

Solution	
	Let x = the actual measurement.
Use an absolute value inequality to express this situation.	|actual-ideal| ≤ tolerance $\|x - 60\| \le 0.075$
Rewrite as a compound inequality.	$-0.075 \le x - 60 \le 0.075$
Solve the inequality. Add each side of the equation by 60.	$59.925 \le x \le 60.075$
Answer the question.	The diameter of the rod can be between 59.925 mm and 60.075 mm.

Notes
Vocabulary: Manufacturing Process (製造過程), Tolerance (容許量), Specification (規格), Exceed (超出), Reject (拒絕), Diameter (直徑), Rod (竿), Vary From (不同於), Acceptable (可接受), Compound (合成).

Sentences:

1. What range of diameters will be acceptable to the customer without causing the rod to be rejected? (客戶能接受的竿子直徑容許範圍為多少？)

2. Rewrite as a compound inequality. (將其重寫成合成不等式。)

References

1. 許志農、黃森山、陳清風、廖森游、董涵冬（2019）。數學 1：單元 2 絕對值。龍騰文化。

2. MathBits Notebook. *Absolute Value.* http://mathbitsnotebook.com/

3. Openstax. *Intermediate Algebra 2e.* https://reurl.cc/GXbylv

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