

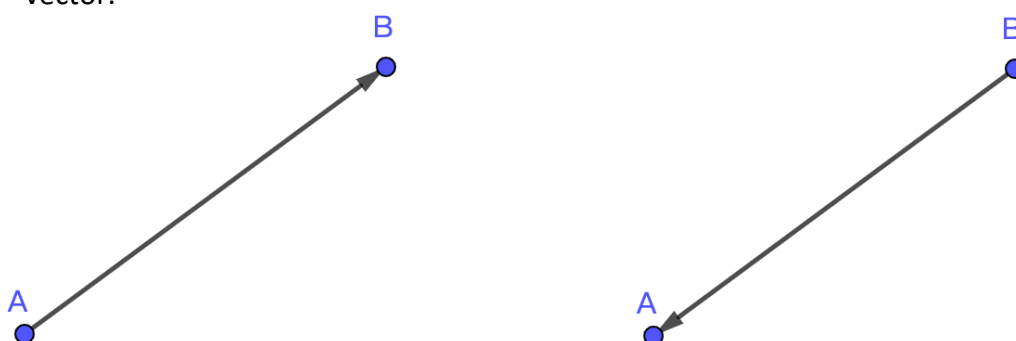
Vectors

I. Key mathematical terms

Terms	Symbol	Chinese translation
vector		
magnitude		
absolute value		

II. Vectors

In mathematics, a vector has a magnitude () and a direction (). A vector carries a point from the initial point to the terminal point. We can picture a vector as a segment with an arrow. The length of the segment is the magnitude of the vector and the arrow indicates the direction of the vector.



This is vector AB . It starts at point A and finishes at point B . A is the initial point and B is the terminal point.

This is vector BA . It starts at point B and finishes at point A . B is the initial point and A is the terminal point.

In physics, the vector is usually compared with the scalar. ()
 A scalar is a physical quantity that is only described by its magnitude.
 For example, mass (), volume(), length(), speed(),
 time() and temperature() are quantities of scalar.

Vector, in physics, is a quantity that has a magnitude and a direction.
 For example, displacement (), velocity(), acceleration()
 and force() are quantities of vectors.

<key> If $\vec{PQ} = \vec{RS}$ then we can say these two vectors have the same magnitude and direction. We can say these two vectors are equal.

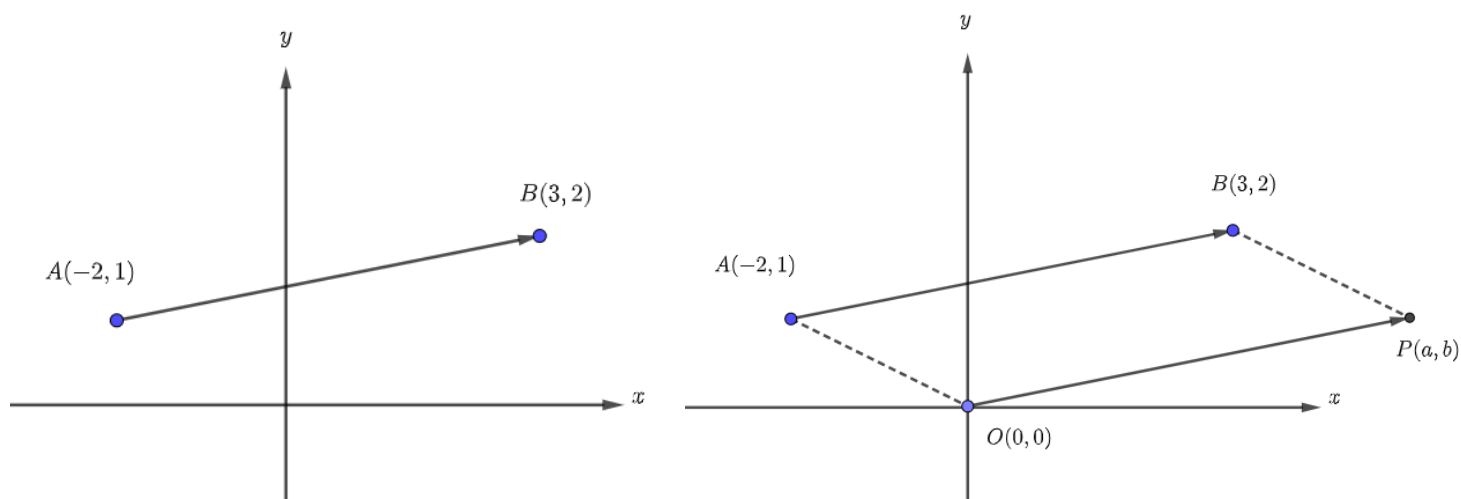
<key> If a vector has the same initial point and terminal point, we call this vector a zero vector or a null vector. This vector has no magnitude and no direction.

Representation of vectors

We can use an arrow to represent a vector. The head of the arrow denotes the direction of the vector. We can use different ways to represent a vector. Let us learn more about the representation of vectors with some examples.

A vector that starts from point A and ends at point B is denoted by \vec{AB} . We can also represent this vector by $-\vec{BA}$ which is the negative of the vector \vec{BA} . Sometimes we use \vec{v} to denote a vector on the plane. That means we cannot identify the initial point and the terminal point of the vector.

To describe a vector on the coordinate plane, we use the component form of a vector. A vector whose initial point is the origin $O(0,0)$ can be uniquely represented by the coordinates of its terminal point $P(a,b)$. The component form of the vector that is being directed from point O to point P can be represented as $\vec{OP} = (a,b) - (0,0) = (a-0, b-0) = (a,b)$. If both the initial point and the terminal point lie at the origin, then this vector is the zero vector and is denoted by $\vec{0} = (0,0)$.



For any vector on the coordinate plane, we can move this vector and let the initial point be the origin to find the component form of the vector. (See figure above.) The component form from A to B is $\vec{AB} = \vec{OP} = (3 - (-2), 2 - 1) = (5, 1) = (a, b)$. a is the x -component (horizontal component) and b is the y -component (vertical component).

If we want to know the magnitude (or length) of a vector, we use the absolute value. The magnitude of vector AB is the length of the vector and is denoted by $|\vec{AB}| = \sqrt{a^2 + b^2} = \overline{AB}$.

Component form of a Vector

If we know the position of two points on the coordinate, we can use the formula to determine the component form of the vector between these two points.

Consider a point $A(x_1, y_1)$ and another point $B(x_2, y_2)$.

1. The vector from A to B is: $\vec{AB} = (x_2 - x_1, y_2 - y_1)$
2. The vector from B to A is: $\vec{BA} = (x_1 - x_2, y_1 - y_2) = -\vec{AB}$
3. Magnitude of vector \vec{AB} is: $|\vec{AB}| = |(x_2 - x_1, y_2 - y_1)| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Example 1.

Consider the points $A(1, 2), B(-2, 3), C(5, 7)$. Find vectors:

(1) \vec{AB}

(2) \vec{BA}

(3) \vec{BC}

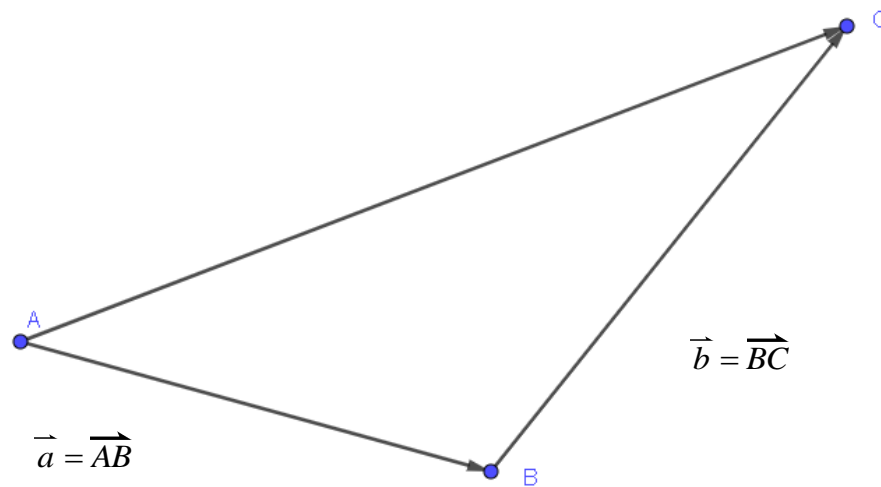
(4) \vec{AC}

(5) $|\vec{AB}|$

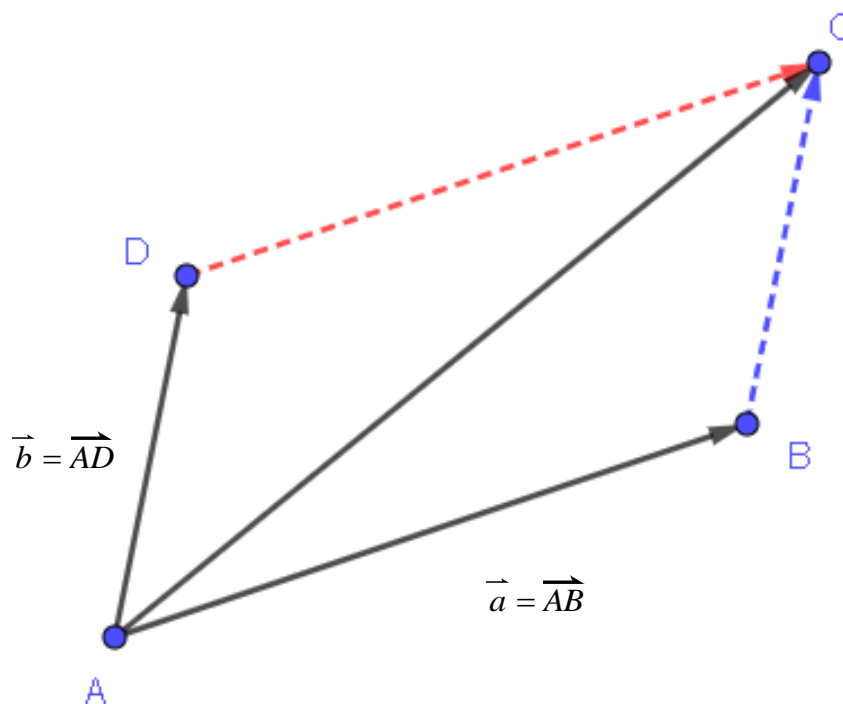
<key> Compare the vectors \vec{AB} , \vec{BC} and \vec{AC} , what do you find?

Vector Operations-vector addition:

We can add two vectors by using the triangle law or parallelogram law. A vector can be considered as an order of motion. For example, we have vectors $\vec{a} = \vec{AB}$ and $\vec{b} = \vec{BC}$. The sum $\vec{AB} + \vec{BC}$ is \vec{AC} . (Vector AC is called the resultant of vectors AB and BC. See figure below. It's just like we order a point to move from point A to point B and then move to point C.) This is the triangle law for vector addition.



Triangle law for vector addition

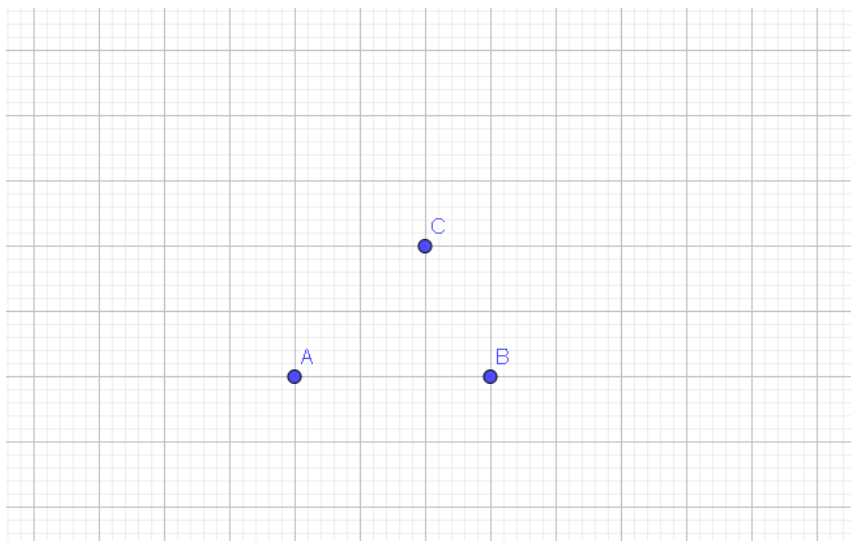


Parallelogram law for vector addition

Example2.

Draw the vectors in the following graph:

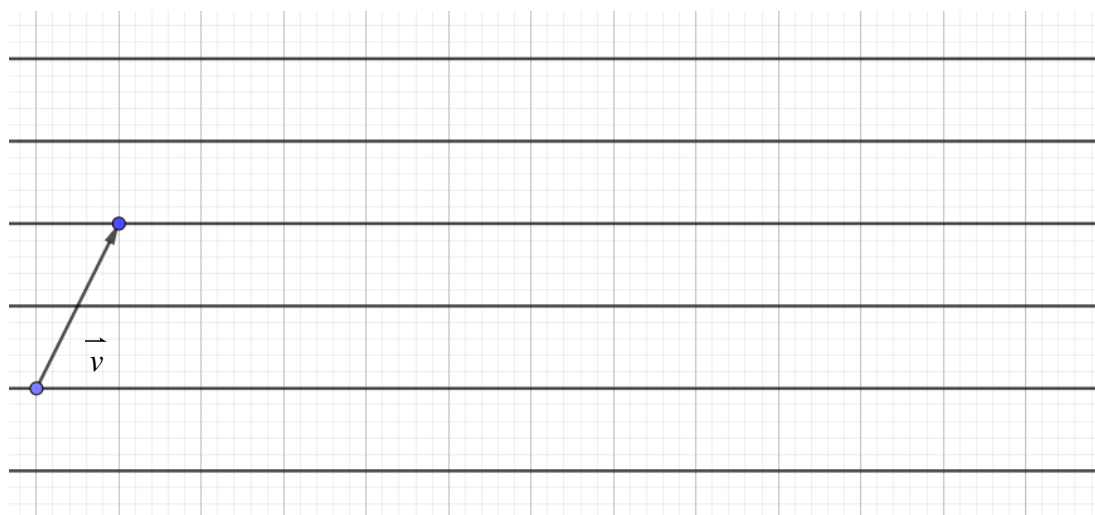
- (1) \vec{AB} (2) $\vec{AB} + \vec{AC}$ (3) $-\vec{AB}$ (4) $\vec{AB} + \vec{BC}$ (5) $\vec{AB} - \vec{BC}$

**Vector Operations-scalar multiplication:**

Scalar multiplication is another basic vector operation. In high school, scalars will always be real numbers. The product of a vector AB and a scalar c is the vector that is $|c|$ times as long as \vec{AB} . If c is positive, $c\vec{AB}$ has the same direction as \vec{AB} , and when c is negative, $c\vec{AB}$ has the direction opposite to the direction of \vec{AB} .

Example3.

Vector v is given in the following graph, draw the vectors with scalar multiplication:



(1) $\frac{1}{2}\vec{v}$

(2) $2\vec{v}$

(3) $-\vec{v}$

(4) $-\frac{3}{2}\vec{v}$

Definition of Vector Operations

Let $\vec{u} = (x_1, y_1)$, $\vec{v} = (x_2, y_2)$ are vectors and c be a scalar. (a real number)

vector addition

$$\vec{u} + \vec{v} = (x_1 + x_2, y_1 + y_2)$$

scalar multiplication

$$c\vec{u} = (cx_1, cy_1)$$

Example 4.

Let $\vec{v} = (-2, 3)$, $\vec{u} = (3, 5)$. Find each vector.

(1) $\vec{v} + \vec{u}$

(2) $2\vec{v} - \vec{u}$

(3) $\frac{1}{2}\vec{v} - \frac{2}{3}\vec{u}$

Properties of Vector Addition and Scalar Multiplication

Let u, v and w are vectors and c, d are scalars. The following properties are true:

1. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ (commutative)

2. $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$ (associative)

3. $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$

4. $\vec{u} + (-\vec{u}) = \vec{0}$

5. $c(d\vec{u}) = d(c\vec{u}) = (cd)\vec{u}$

6. $(c+d)\vec{u} = c\vec{u} + d\vec{u}$ (distributive)

7. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$

8. $|c\vec{v}| = |c||\vec{v}|$

<key>

You must remember all the properties mentioned above. We'll use these properties

when we want to do the vector operations.

Example 5.

Let $\vec{u} = (1,3)$, $\vec{v} = (-2,1)$, $\vec{w} = (0,1)$. Find the following:

(1) $3(\vec{v} + \vec{u})$ (2) $(\vec{v} + \vec{w}) + \vec{u}$ (3) $\left| \vec{v} - \vec{u} \right|$

* (4) $\frac{\vec{v}}{\left| \vec{v} \right|}$ * (5) $\left| \frac{\vec{v}}{\left| \vec{v} \right|} \right|$

<key> Unit vector is a vector of magnitude(length) 1.

<資料來源>

1. Vectors

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<https://www.mathsisfun.com/algebra/vectors.html>

<https://www.cuemath.com/geometry/vectors/>

2. Ron Larson, Precalculus, International Metric Edition

3. Pearson Edexcel AS and A level Mathematics Pure Mathematics Year 1/AS Chapter

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