Topic: Exponential Functions and Their Graphs

1. (Let's start with an example!)

The temperature, T, of a cup of tea is modeled by the function $T(x) = 21 + 55(1.9)^{-x}$, where T is measured in degrees Celsius after the tea is poured into the cup for x minutes. Answer the following questions with a calculator and Desmos:

a. Complete the table

x	20	40	60	80	100
T					

b. How long did it take for the temperature to drop from 100 degrees to 80 degrees?

c. How long did it take for the temperature to drop from 80 degrees to 60 degrees?

d.How long did it take for the temperature to drop from 60 degrees to 40 degrees?

e.How long did it take for the temperature to drop from 40 degrees to 20 degrees?

f. Use Desmos to plot the graph of the function. Is it a linear function? State your reason.

g.Will the temperature of the tea ever reach $0°C$? State your reason.

2. The function of the example is an exponential function, one in which the independent variable is the exponent of a number.

Definition of Exponential Function
The exponential function f with base a is denoted by
$$f(x) = k \times a^x + c$$
where $a > 0, a \neq 1$, and x is any real number, k and c are constant.
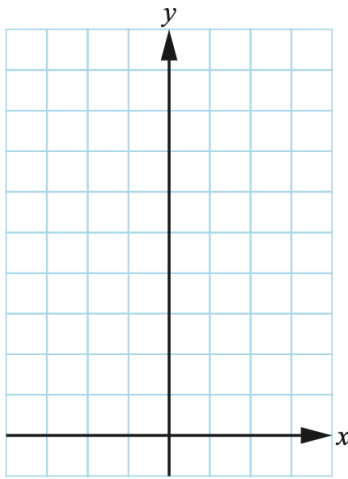

The simplest form of an exponential function is $f(x) = a^x$, $a > 0$, $a \neq 1$.

3. Graphs of exponential functions

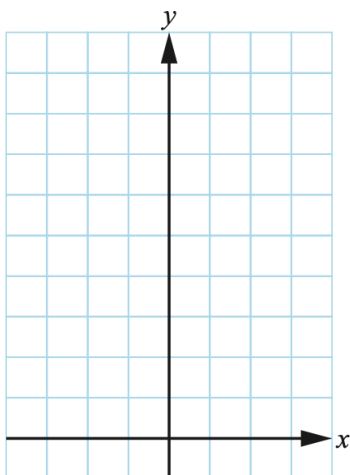Complete the following table, plot the points from the table, and connect them with a smooth curve.

x	−2		0	1	2	3
y		$\dfrac{1}{2}$				

a. $f(x) = 2^x$

x	−2	−1	0	1	2
y					

b. $f(x) = 3^x$

x	−2	−1	0	1	2
y					

c. $f(x) = (\dfrac{1}{2})^x$
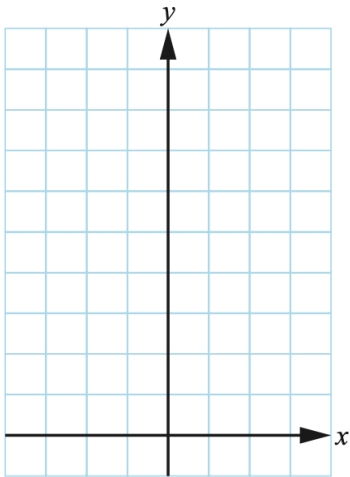
| x	−2	−1	0	1	2
y					

d. $f(x) = (\dfrac{1}{3})^x$
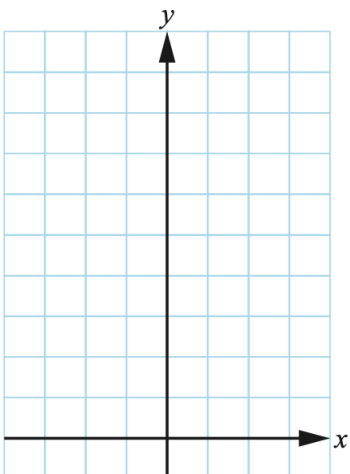
e. What are the similarities and differences among these graphs?

4. 在 Desmos 中依序輸入下列函數及選擇新增滑桿 a (可由教師輸入，亦可選擇其他軟體操作): $f_1(x) = a^x$, $f_2(x) = (\frac{1}{a})^x$ 拉動滑桿觀察圖形變化並回答下列問題:

a. How can you describe the shape of these graphs? Do they have any asymptotes? Are they increasing or decreasing?

b. Give two different conditions on the base and the exponent for an exponential function to be a decreasing function.

c. What happens to the y–value as x approaches positive infinity or negative infinity?

d. How many zeros do these functions have and why?

e. Describe the main features of the graphs of all exponential functions of the form $f(x) = a^x$, $a > 0$, $a \neq 1$

5. Conclusion (參考資料：Ron Larson, Precalculus with Limits 4th edition)

Consequently, the graph of F is a reflection (in the y-axis) of the graph of f. The graphs of G and g have the same relationship. The graphs in Figures 3.1 and 3.2 are typical of the exponential functions $y = a^x$ and $y = a^{-x}$. They have one y-intercept and one horizontal asymptote (the x-axis), and they are continuous. Here is a summary of the basic characteristics of the graphs of these exponential functions.
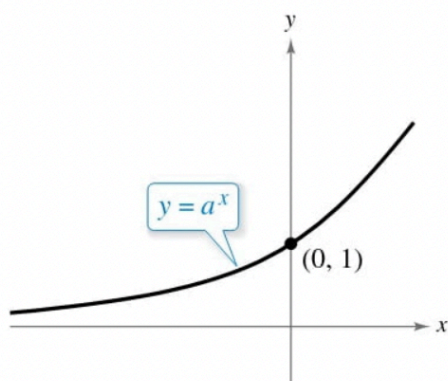
Figures 3.1

Graph of $y = a^x$, $a > 1$
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- y-intercept: $(0, 1)$
- Increasing
- x-axis is a horizontal asymptote ($a^x \to 0$ as $x \to -\infty$).
- Continuous
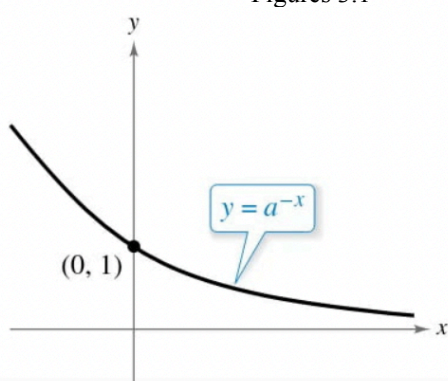
Figures 3.2

Graph of $y = a^{-x}$, $a > 1$
- Domain: $(-\infty, \infty)$
- Range: $(0, \infty)$
- y-intercept: $(0, 1)$
- Decreasing
- x-axis is a horizontal asymptote ($a^{-x} \to 0$ as $x \to \infty$).
- Continuous

Topic: Exponential Functions and Their Graphs 及使用建議

內容	1.（Let's start with an example!） ....以下省略
使用建議	[教學活動安排] 使用一個指數函數的例子及提問讓學生探索其函數式及圖形。 [可參考的英文問句/提問/開場] Today we're going to learn the exponential functions and their graphs. So, what are exponential functions and their graphs? How do we apply it? Let's start with an example. OO(點一個同學) Read it aloud the example.Good. Thank you. Finish the questions in this example with a calculator. We will discuss it in five minutes.
內容	2. The function of the example ....以下省略
使用建議	[教學活動安排] 介紹指數函數的定義 [可參考的英文問句/提問/開場] When you finish the table of the example, the x you plug number in is the exponent or we call the power. So the general form of the exponential function is $f(x) = k \times a^x + c$ (f of x equals k times a to the power of x plus c). k and c are constant , with the restriction that a is greater than zero, not equal to one. The simplest form of an exponential function is $f(x) = a^x, a > 0, a \neq 1.$
內容	3. Graphs of exponential functions ....以下省略

使用建議	[教學活動安排] 用4個簡單的例子讓學生帶點徒手畫去感覺指數函數圖形的長相 [可參考的英文問句/提問/開場] Let's look at the properties of all exponential ~~functions~~ graphs. Let's sketch the function $f(x) = 2^x$ (f of x equals to two to the power of x) together. First, let's fill in the table. Second, plot the points from the table, and connect them with a smooth curve. Now move on to the next part. Sketch the rest functions on your own. We will check the answer in three minutes! Observe these four functions and their graphs. What are the similarities and differences among these graphs? Any volunteers want to share? 給學生的回饋：Good! Thank you for sharing! 在剛剛第3題，我們觀察到指數函數圖形有些共同的特徵，有些不一樣的特徵，那我們就利用Desmos來確認。（這句英文要怎麼翻？）（上課適合講嗎？） Let's move on to next part.
內容	4. 在 Desmos 中依序輸入....以下省略
使用建議	[教學活動安排] 銜接3,學生會觀察到一些指數函數圖形的特徵。因此在這裡利用Desmos有滑桿的功能及搭配提問讓學生去更仔細的觀察。 [可參考的英文問句/提問/開場] Let's use Desmos to help us observe more about the graphs of exponential functions. Follow the instructions on the worksheet and answer the questions.
內容	5. Conclusion....以下省略。

使用建議	[教學活動安排] 為指數函數圖形的特徵下結論。 [可參考的英文問句/提問/開場] Let's sum up the properties we have found for the graphs of exponential functions. Let's read this and check what we found.(我們一行一行的看,來看我們有沒有剛剛在前面是不是就有觀察到了!)

製作者:台北市立育成高中 林玉惇老師