雙語教學主題(國中九年級上學期教材): 三角形的外心
Topic: introducing the circumcenter of a triangle

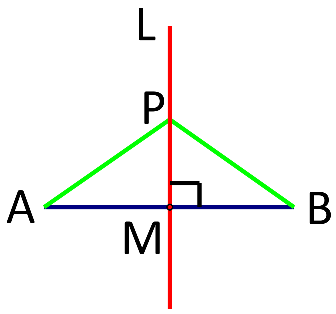
The teaching materials for introducing circles in our textbooks have been changed quite a bit due to the **108 syllabus.** Therefore, the content here is based on the official textbooks-NANI, KANG HSUAN and HANLIN.
由於 108 新綱教材大改，所以這個單元參考 108 新課綱及南一、康軒及翰林版國中數學課本第五冊

Vocabulary
circumcenter, perpendicular bisector, circumcircle, vertex, vertices, congruent, equidistant,

Before we start introducing this new lesson, we definitely need to review some of the related geometry properties.

Review :
Property of perpendicular bisector:
As the figure shown on the right side, line L is the perpendicular bisector of segment AB and intersects $\overline{AB}$ at point M.
Point P is any point on line L, $\overline{PM} \perp \overline{AB}$, $\overline{MA} = \overline{MB}$
Connect $\overline{PA}$ and $\overline{PB}$, then

$$\overline{PA} = \overline{PB}$$



Please review the the proof on your own.
(Hint: you can use symmetry, triangle congruence or Pythagorean theorem, etc.)

The inverse of **Property of perpendicular bisector** is also true. That is:
As shown in figure 2, $\overline{PA} = \overline{PB}$
Then if $\overline{PM} \perp \overline{AB}$ or $\overline{PM}$ is the bisector of $\overline{AB}$
By using triangle congruencey(RHS or SSS), we can easily get $\overline{PM}$ is the perpendicular bisector of $\overline{AB}$.

So the converse is true

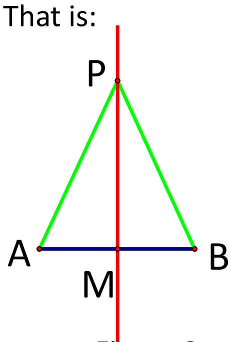


Figure 2

Circumcenter:

First, let's see how it's formed in an acute triangle.

In figure 1, ΔABC is an acute triangle. Line L is a perpendicular bisector of side BC and intersects segment BC at point D, line M is a perpendicular bisector of side AC and intersects segment AC at point E. Line L and M intersect each other at point O.

Then point O is the circumcenter of ΔABC.

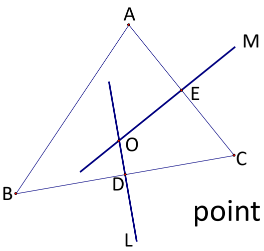


Figure 1

There's a lot to talk about when it comes to a circumcenter. We'll discuss all the important related properties, and we will understand the reason of the name-circumcenter.

Will three perpendicular bisectors of three sides of a triangle intersect at the same point we just mentioned in firgure 1?

The answer is yes. Let's prove it!.

Pf:

From review 1

Line L and line M are perpendicular bisectors of sides $\overline{BC}$ and $\overline{AB}$ respectively and they intersect each other at point O.

Then $\overline{OB}=\overline{OC}$, and $\overline{OC}=\overline{OA}$ $\Rightarrow$ $\overline{OB}=\overline{OA}$

Let line N be perpendicular to segment AB through point O and interescts segment AB at point F.

As shown in figure 2

We can easily prove that $\Delta AOF \cong \Delta BOF$

$\overline{OB}=\overline{OA}$, $\overline{OF}=\overline{OF}$, $\angle AFO=\angle BFO=90^\circ$

$\Delta AOF \cong \Delta BOF$(RHS)

So $\overline{AF}=\overline{BF}$ (the corresponding sides are congruent)

Since line N($\overleftrightarrow{OF}$) is perpendicular to segment AB ($\overleftrightarrow{OF} \perp \overline{AB}$)

and $\overline{AF}=\overline{BF}$, line N($\overleftrightarrow{OF}$) is a perpendicular bisector of segment AB.

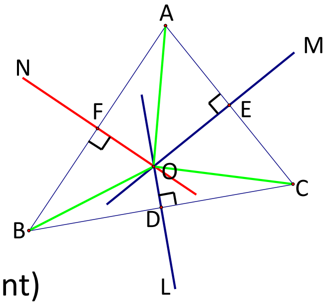


Figure 2

if's proved that in an acute triangle, the perpendicular bisectors of three sides intersect at one point. They all intersect at point O.

in figure 2, we can also see another fact that $\overline{OC}=\overline{OB}=\overline{OA}$, means if we take point O as a center, $\overline{OA}$ as the radius, the circle O will pass through the vertices of ΔABC, then circle O is the circumcircle of ΔABC. That tells why we name point O circumcenter. It also means the circumcenter of a triangle is equidistant to the vertices.

The conclusion that:

"Three perpendicular bisectors of three sides of a triangle intersect at one point, circumcenter. "

Is not only true in acute triangles. we can do similar proofs for a right triangle and an obtuse triangle. So the conclusion is true in any kind of triangles. Next, let's take a look of the position of the circumcenter in acute triangles, right triangles, and obtuse triangles..

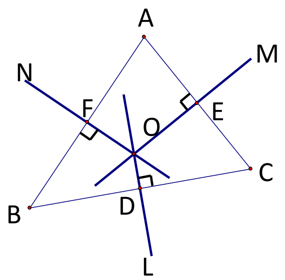


Figure 4

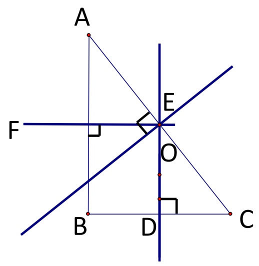


Figure 5

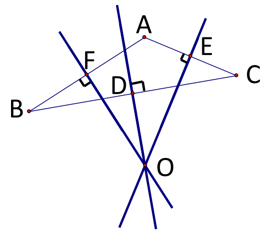
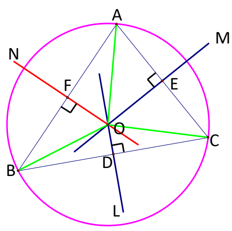
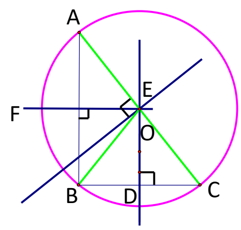
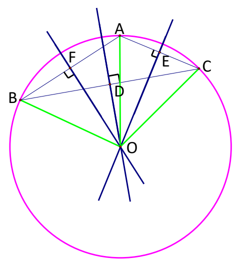


Figure 6

Position of the circumcenter in different kind of triangles；

Acute triangle	Right angle triangle	Obtuse triangle
interior of the triangle	On the hypotenuse	Exterior of the triangle
	Actually, point O is on the **midpoint** of the hypotenuse	

And the circumcircle of each triangle is shown as follows.

Please construct the circumcenter in each kind of triangles with your compasses and rulers to check the results mentioned above.

Now I want to ask you a question:
How do we know that in a right triangle, the circumcenter is on its hypotenuse? Furthermore, the circumcenter is actually the midpoint of the hypotenuse! It’s not easy to prove that a point is on a line. Here we do the reverse attempt.

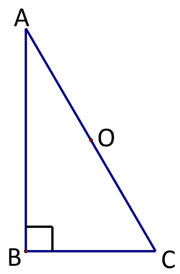


Figure 1

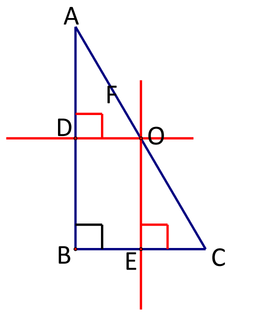


Figure 2

In figure 1, ΔABC is a right triangle, $\angle B=90^\circ$. Point O is the midpoint of segment AC.

We construct $\overline{OD}\perp\overline{AB}$ and $\overline{OD}$ intersects $\overline{AB}$ at point D.
$\overline{OE}\perp\overline{BC}$ and $\overline{OE}$ intersects $\overline{BC}$ at point E.

Then we get $\overline{OD}//\overline{BC}$ (for $\overline{OD}$ and $\overline{BC}$ are perpendicular to the same segment AB)

In ΔABC, when $\overline{OD}//\overline{BC}$, $\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AO}}{\overline{CO}}$

But point O is the midpoint of segment AC, then $\frac{\overline{AD}}{\overline{BD}}=\frac{\overline{AO}}{\overline{CO}}=1$, it means $\overline{AD}=\overline{BD}$, point D is the midpoint of segment AB.

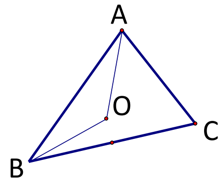
We prove that $\overline{OD}$ is the perpendicular bisector of side AB of ΔABC.
Similarly $\overline{OE}$ is the perpendicular bisector of side AC of ΔABC.
From the previous discussion, we conclude that
the midpoint O of the hypotenuse AC is the circumcenter of ΔABC.
This is a very important property of a right triangle. Please study it again and again.
And for those who are interested, think of other ways to prove the this property.

We need to do some practice after learning so many properties of circumcenter.

Ex 1:

Point O is the circumcenter of ΔABC, $\angle C=75^\circ$

Find the measure of $\angle AOB$

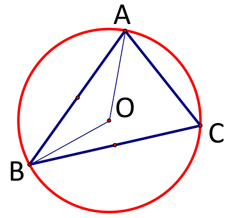


Sol:

let's construct the circumcircle of ΔABC, then

$\angle AOB$ is a central angle and $\angle ACB$ is the inscribed angle of circle O.

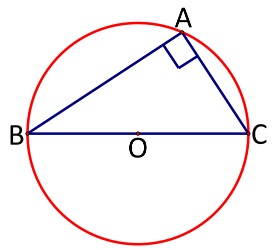
therefore, $\angle AOB=2\angle ACB=150^\circ$ #



Ex 2:

As shown in the figure, ΔABC is a right triangle, $\angle A=90^\circ$, point O is the midpoint of segment BC, Circle O is the circumcircle of ΔABC.

If $\overline{AB}=8$, $\overline{AC}=6$, please find the area of circle O.



Sol:

Point O is the circumcenter of ΔABC,

so the length of the radius segment OB= $\frac{1}{2}$ segment BC.

And $\overline{BC}^2=\overline{AB}^2+\overline{AC}^2$

$=8^2+6^2$

$=100$

$\overline{BC}=10$

So the area of circoe O= $\pi\cdot(\frac{1}{2}\cdot 10)^2=25\pi$ #

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