# Workshop on Optimization 

November 29-30, 2005

Department of Mathematics<br>National Taiwan Normal University<br>Taipei 11677, Taiwan

Sponsored by
College of Science, National Taiwan Normal University
Department of Mathematics, National Taiwan Normal University
Mathematics Research Promotion Center, Taiwan

Organized by
Liang-Ju Chu and Jein-Shan Chen

## Schedule of Programs

Place: M106, Mathematics Building

| November 29 (T) | Speakers/Events | November 30 (W) | Speakers/Events |
| :---: | :---: | :---: | :---: |
| $09: 00-09: 30$ | Registration | $09: 30-10: 10$ | Wataru Takahashi |
| $09: 30-10: 10$ | Liqun Qi | $10: 10-10: 50$ | Nobuo Yamashita |
| $10: 10-10: 50$ | Hidefumi Kawasaki | $10: 50-11: 10$ | BREAK |
| $10: 50-11: 10$ | BREAK | $11: 10-11: 45$ | Yuh-Jye Lee |
| $11: 10-11: 45$ | Ruey-Lin Sheu | $11: 45-12: 20$ | Mau-Hsiang Shih |
| $11: 45-12: 20$ | Chih-Jen Lin |  |  |
| $12: 20-14: 10$ | LUNCH |  |  |
| $14: 10-14: 45$ | Sy-Ming Guu |  |  |
| $14: 45-15: 20$ | Bertrand M.T. Lin |  |  |
| $15: 20-15: 40$ | BREAK |  |  |
| $15: 40-16: 15$ | Lai-Ju Lin |  |  |
| $16: 15-16: 50$ | Shunsuke Hayashi |  |  |
| $17: 30$ | DINNER |  |  |

# Minimizing a linear function under max-min type constraints 

Sy-Ming Guu<br>Department of Business Administration<br>Yuan Ze University<br>Tao-Yuan, Taiwan<br>iesmguu@saturn.yzu.edu.tw


#### Abstract

In this talk, we investigate the problem of minimizing a linear objective function subject to max-min type constraints. As an application, such model has been applied for the streaming media provider seeking a minimum cost while fulfilled the requirements assumed by a three-tier framework. We shall propose a necessary condition for optimal solutions to hold. Based on this necessary condition, we propose a branch-and-bound procedure to solve the problem. Experimental results show that our procedure systematically outperforms existing algorithms.


# Tone-monopolistic approaches for sum rate maximization of digital subscriber lines 

Shunsuke Hayashi<br>Graduate School of Informatics<br>Kyoto University<br>Yoshida-hommachi, Sakyoku<br>Kyoto, Japan<br>hayashi@amp.i.kyoto-u.ac.jp


#### Abstract

We deal with the sum rate maximization problem for digital subscriber line (DSL) systems. In this problem, the objective function is written as the the sum of logarithms of fractions. Since the objective function is non-concave, there exists many local maxima. Several approaches have been proposed for solving the sum rate maximization problem. One of the most popular algorithms is the iterative water-filling algorithm (IWFA). This algorithm is based on game theoretical ideas, and attains a good solution if the crosstalk is small enough. On the other hand, it is also known that IWFA does not work well when the crosstalk coefficients are large.

In this study, we show that, if the crosstalk coefficients are large, then the optimal solution has the tone-monopolistic structure, which means every tone is used by only one user. Moreover, we also show that, even if the crosstalk is not so strong, there exists a tone-monopolistic local optimum under appropriate conditions. We also give algorithms which solve sum rate problem under assumption that the optimal solution is tone-monopolistic. Our algorithm is based on greedy method. The numerical results indicate that our algorithm calculates a solution in a sufficiently short time, and shows better performance than IWFA, if crosstalk coefficients and power budget are large enough.


# Duality theorems based on triangles separating three convex sets 

Hidefumi Kawasaki<br>Department of Mathematical Sciences<br>Kyushu University<br>Fukuoka 812-8581, Japan<br>kawasaki@math.kyushu-u.ac.jp


#### Abstract

The separation theorem asserts that two disjoint convex sets can be separated by a hyperplane, and it plays the central role in duality theory in optimization. On the other hand, the three-phase partition problem is to divide a given domain in $R^{2}$ into three sub-domains with a triple junction having least interface area. The aim of this talk is to give duality theorems for optimization problems induced from the three-phase partition problem. For this aim, we introduce a new concept of separation of three convex sets by a triangle.


# Smooth Support Vector Machines for Classification and Regression 

Yuh-Jye Lee<br>Department of Computer Science and Information Engineering<br>National Taiwan University of Science and Technology<br>Taipei, Taiwan<br>yuh-jye@mail.ntust.edu.tw


#### Abstract

Support Vector Machines (SVMs), a new generation learning system based on the statistical learning theory, have been significantly developed in the theoretical understanding as well as algorithmic implementing in the last decade. In this talk, we employ a smoothing technique to reformulate the support vector machines for classification and regression as an unconstrained minimization problem. We term such reformulation Smooth Support Vector Machines (SSVMs). A fast Newton-Armijo algorithm for solving SSVMs is proposed that converges to the unique solution of the unconstrained minimization problem globally and quadratically. In order to deal with the nonlinear classification and regression problems with massive data sets, we introduce the reduced kernel technique that will speed up the computational time as well as reduce the memory usage. Numerical results and comparisons are given to demonstrate the effectiveness and speed of SSVMs


# Optimization issues in training support vector machines 

Chih-Jen Lin<br>National Taiwan University<br>Taipei, Taiwan<br>cjlin@csie.ntu.edu.tw


#### Abstract

Optimization plays an important role in many machine learning methods. However, the gap between the two areas is large as as traditionally they have very different focuses. In this talk, I will try to bridge the two areas by using support vector machines (SVM) as an example. SVM has been a promising machine learning method which involves quite a few challenging optimization problems. We will particularly discuss the large dense quadratic programming problems from SVM.


# Existence Theorems for Bilevel Problems with Applications to Mathematical Programs with Equilibrium Constraints and Semi-Infinite Problems 

Lai-Ju Lin<br>Department of Mathematics<br>National Changhua University of Education<br>Changhua, 50058 Taiwan<br>maljlin@math.ncue.edu.tw


#### Abstract

In this paper, we establish existence theorems for bilevel problems. The aim of this paper is to investigate under which conditions the existence of feasible points of bilevel problem can be assumed in advanced and which conditions there existence minimizers of this type of problem. From which we establish existence theorems of mathematical programs with equilibrium constraint and semi-infinite problem.


# Scheduling Problems with Time-Dependent Processing Times 

Miao-Tsong Lin<br>Institute of Information Management<br>National Chiao Tung University<br>Hsin-Chu, Taiwan<br>bmtlin@mail.nctu.edu.tw


#### Abstract

In the literature, parameters involved in most deterministic optimization problems are assumed to be fixed once given. However, there are applications in which the parameters may vary. Machine scheduling problems with time-/positiondependent processing times have received increasing attention in recent years. In this talk, we discuss some scheduling problems involving time-dependent processing times. Our results include NP-hardness proofs, dynamic programming algorithms, approximation algorithms. We will discuss some questions that are still open.


# Eigenvalues of Tensors and Their Applications 

Liqun Qi<br>Department of Applied Mathematics<br>The Hong Kong Polytechnic University<br>Hong Hom, Hong Kong<br>maqilq@polyu.edu.hk


#### Abstract

A tensor is represented by a supermatrix under a co-ordinate system. Motivated by the positive definiteness issue in automatic control, I defined eigenvalues and eigenvectors of a real completely symmetric supermatrix, and explored their practical applications in judging positive definiteness of an even degree multivariate form.


However, the tensor studied in nonlinear continuum mechanics and physics are physical quantities which are invariant under co-ordinate system changes. In particular, the coefficients of the characteristic polynomial of a second order tensor are principal invariants of that tensor. Motivated by this, we defined E-eigenvalues and E-eigenvectors for tensors. The E-eigenvalues of a tensor are the same as the E-eigenvalues of its representation supermatrix in an orthonormal co-ordinate system. Based upon the resultant theory, we define the E-characteristic polynomial of an $m$ th order $n$-dimensional tensor. An E-eigenvalue of a tensor is a root of the E-characteristic polynomial. In the regular case, a complex number is an Eeigenvalue if and only if it is a root of the E-characteristic polynomial. We convert the E-characteristic polynomial of a tensor to a monic polynomial and show that the coefficients of that monic polynomial are invariants of that tensor, i.e., they are invariant under co-ordinate system changes. We call them principal invariants of that tensor. The maximum number of the principal invariants of an $m$ th order $n$ dimensional tensor is a function of $m$ and $n$. We denote it by $d(m, n)$ and show that $d(1, n)=1, d(2, n)=n, d(m, 2)=m$ for $m \geq 3$ and $d(m, n) \leq m^{n-1}+\cdots+m$ for $m, n \geq 3$.

Independently, Gene Golub and his student Lek-Heng Lim are also exploring the definition and applications of eigenvalues of tensors. This echoes our efforts and stimulates further exploration in this new territory.

# Performance Estimations of First Fit Algorithm for Online Bin Packing with Variable Bin Sizes and LIB constraints 

Ruey-Lin Sheu<br>Department of Mathematics<br>National Cheng Kung University<br>Tainan, Taiwan<br>rsheu@mail.ncku.edu.tw


#### Abstract

We consider the NP Hard problem of online Bin Packing while requiring that larger (or longer) items be placed below smaller (or shorter) items - we call such a version the LIB version of problems. Bin sizes can be uniform or variable. We provide analytical upper bounds as well as computational results on the asymptotic approximation ratio for the first fit algorithm.


# Stabilization of Gene Expression and Negative Feedback Loops 

Mau-Hsiang Shih<br>Department of Mathematics<br>National Taiwan Normal University<br>Taipei 11677, Taiwan<br>mhshih@math.ntnu.edu.tw


#### Abstract

A principal aim of modern molecular biology is to elucidate the nature of development by exploring the dynamics of gene regulatory networks at the systems level. A gene, when expressed, leads to the production of proteins that can act to either activate or inhibit the expression of other genes. Within networks of such interacting genes and proteins, feedback loops play a central role in controlling the dynamics, and biologists have for many years recognized that the existence of a negative feedback loop is necessary ingredient for homeostasy. In this talk, we will present a mathematical proof and we wish to suggest that the biologist's intuition is probably correct.


# Fixed Point Algorithms in Optimization 

Wataru Takahashi<br>Department of Mathematical and Computing Sciences<br>Tokyo Institute of Technology<br>Tokyo 152-8552, Japan<br>wataru@is.titech.ac.jp


#### Abstract

We first discuss iterative methods for approximation of fixed points of non-expansive mappings in Hilbert spaces. Next, we give proximal point methods for finding zero points of maximal monotone operators in Hilbert spaces. Further, we extend these results to Banach spaces. Using these results, we discuss the problem of finding minimizers of convex functions and the problem of finding saddle points of two variable functions. Finally, we deal with projection methods for approximation of solutions of variational inequalities of nonlinear operators.


# Sparse quasi-Newton updates with positive definite matrix completion 

Nobuo Yamashita<br>Graduate School of Informatics<br>Kyoto University<br>Yoshida-hommachi, Sakyoku<br>Kyoto, Japan<br>nobuo@i.kyoto-u.ac.jp


#### Abstract

The quasi-Newton method is a powerful method for solving unconstrained minimization problems. However, since the approximate Hessian generated by the usual quasi-Newton update (e.g. BFGS or DFP) becomes dense, the quasi-Newton method cannot be applied for large-scale problems due to lack of memory. To overcome this difficulty, we propose sparse quasi-Newton updates with positive definite matrix completion that exploit the sparsity pattern $E:=\left\{(i, j) \mid\left(\nabla^{2} f(x)\right)_{i j} \neq\right.$ 0 for some $\left.x \in R^{n}\right\}$ of the Hessian. The proposed method first calculates a partial approximate Hessian $H_{i j}^{Q N},(i, j) \in F$, where $F \supseteq E$, by using an existing quasi-Newton update formula such as BFGS or DFP. Next, we obtain a full matrix $H_{k+1}$, which is a maximum-determinant positive definite matrix completion of $H_{i j}^{Q N},(i, j) \in F$. If the sparsity pattern $E$ (or its extension $F$ ) has a property related to a chordal graph, then the matrix $H_{k+1}$ can be expressed as products of some sparse matrices. Therefore, if the Hessian is sparse, the time and space complexities of the proposed method are far fewer than those of the BFGS or the DFP. In particular, when the Hessian matrix is tridiagonal, the complexities become $O(n)$. We show that the proposed method has superlinear convergence under the usual assumptions.


