2006『最佳化理論及其應用』工作坊

Workshop on Optimization, 2006

國立台灣師範大學數學系

December 8-10, 2006

Sponsored by

國科會數學推動中心
國立台灣師範大學理學院

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朱亮儒、陳界山

Schedule of Programs
Place : M210, Mathematics Building

Dec. 8 (FRI)	Events	Dec. 9 (SAT)	Events	Dec. 10 (SUN)	Events
		08:30 - 09:00	Registration	8:30 - 9:00	Registration
		09:00 - 09:50	**Plenary talk**	09:00 - 09:50	**Plenary talk**
		09:50 - 10:10	*BREAK*	09:50 - 10:10	*BREAK*
		10:10 - 10:35	Wataru Takahashi	10:10 - 10:35	Defeng Sun
		10:35 - 11:00	"	10:35 - 11:00	"
		11:10 - 11:35	Hong-Kun Xu	11:10 - 11:35	Jiming Peng
		11:35 - 12:00	"	11:35 - 12:00	"
		12:00 - 13:30	*LUNCH*	12:00 -	*LUNCH*
13:00 - 14:00	年會報到	13:30 - 13:55	Yasunori Kimura		
14:00 - 14:40	年會開幕	13:55 - 14:20	"		
14: 40 - 15:30	**Plenary talk**	14:20 - 14:45	Shuechin Huang		
15:40 - 16:05	Han-Lin Li	14:45 - 15:05	*TEA BREAK*		
16:05 - 16:30	"	15:05 - 15:30	Yen-Cherng Lin		
16:30 - 16:55	Xin Chen	15:30 - 15:55	Po-Feng Wu		
16:55 - 17:20	"	15:55 - 16:20	Wei-Shih Du		
18:00 - 21:00	*RECEPTION*				

Duality Approach to Inventory Centralization Games

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Abstract. Linear programming (LP) duality has played a fundamental role in the analysis of cooperative games. In this lecture, we will present new applications of LP duality and stochastic LP duality in studying cooperative games arising from inventory centralization. In particular, we show that duality theory can be used to prove the non-emptiness of cores for such inventory games and to find an element in the core.

The first example is the economic lot-sizing game, in which multiple retailers form a coalition by placing joint orders to a single supplier in order to reduce ordering cost, which is assumed to be a concave function of the order quantity. We are concerned with the issue of how to allocate the cost/benefit so that it is advantageous for every retailer to join the coalition. The standard formulation of the corresponding optimization problem is a concave minimization problem and hence LP duality does not directly apply. We suggest an integer programming formulation for this optimization problem and show that its LP relaxation admits zero integrality gap, which makes it possible to analyze the game using LP duality. We show that there exists an optimal dual solution that gives rise to an allocation in the core, which can be found in polynomial time. An interesting feature of our approach is that, in contrast to the duality approach for other known cooperative games, it is not necessarily true that every optimal dual solution gives rise to a core allocation.

Another example is a single-period inventory centralization game with stochastic demand where multiple retailers form a coalition by holding centralized inventory in order to take advantage of the effect of risk pooling. Again, we are concerned with the issue of how to allocate the cost/benefit. When the ordering cost is linear, the optimization problem corresponding to the inventory game is formulated as a stochastic program. We observe that the strong duality of stochastic LP not only directly leads to a series of recent results concerning the non-emptiness of the cores of such games, but also suggests a way to find an element in the core. We further construct a nontrivial infinite dimensional linear programming dual for the well-known newsvendor problem with concave ordering cost and prove a strong duality result for this non-convex minimization problem. This new duality result immediately implies that the corresponding game has a non-empty core. Finally,

we prove that it is NP-hard to determine whether a given allocation is in the core for the newsvendor game even in a very simple setting.

[This is a joint work with Jiawei Zhang at New York University.]

Iterative approximation of fixed points of asymptotically nonexpansive mappings in reflexive Banach spaces

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Abstract. Let X be a reflexive Banach space, C a nonempty closed convex subset of X, $f : C \to C$ a contraction, $\{T_i : C \to C\}_{i=1}^m$ a finite family of asymptotically nonexpansive mappings with sequences $\{k_{i+m(j-1)}\}_j \subset [1, \infty)$ $(1 \le i \le m)$, $\{t_n\}$ a sequence in $(0, 1)$. We will establish the necessary and sufficient conditions for the following iterative sequence to converge to a common fixed point of $T_1, T_2, \cdots, T_m$:

$$z_{n+1} = \left(1 - \frac{t_n}{k_n}\right) f(z_n) + \frac{t_n}{k_n} T_{\bar{n}}^{j_n} z_n, \quad n \in N,$$

where $\bar{n} \equiv \mathrm{mod}\ n$ and $n = \bar{n} + m(j_n - 1)$.

Iterative Methods for a Sequence of Mappings in a Banach Space

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Abstract. Let us consider the problem of finding a solution $z \in E$ of an operator inclusion $0 \in Az$, where A is an accretive operator defined on a real Banach space E. One of the most popular iterative schemes of approximating this solution is the following method called the proximal point algorithm, which was first introduced by Martinet and has been studied by Rockafellar: Let $x_1 \in E$ and generate a sequence $\{x_n\}$ by $x_{n+1} = (I + \rho_n A)^{-1} x_n$ for $n \in N$, where $\{\rho_n\}$ is a sequence of positive real numbers satisfying that $\inf_{n \in N} \rho_n > 0$. Convergence of this scheme has been studied by many researchers with various types of additional conditions. In this talk, we consider this type of iterative schemes and introduce some recent development related to this scheme.

On Maximal Element Theorems, Variants of Ekeland's Variational Principle and Their Applications

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Abstract. In this paper, we establish several different versions of generalized Ekeland's variational and maximal element theorems for τ-functions in $\preceq$ complete metric spaces. The equivalence relations between maximal element theorem, generalized Ekeland's variational principle, generalized Caristi's (common) fixed point theorems and nonconvex maximal element theorems for maps are also proved. Moreover, we obtain some applications to a nonconvex minimax theorem, nonconvex vectorial equilibrium theorems and convergence theorems in complete metric spaces.

Presenter: Wei-Shih Du (杜威仕).

Making Decision on Crystal Balls

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Abstract. This study proposes a visualization method of displaying decision alternatives on spheres . Given a set of alternatives with some attributes, we intend to rank and to group these alternatives based on a decision maker's preferences on the attributes. Following various types of specifying preferences, four models are proposed: Moral Algebra Model, Even Swap Model, Pairwise Comparison Model, and Classification Model . By examining the moving trajectory of each alternative on spheres, a decision maker can adjust his preferences thus to reach a decision more confidently. Some practical examples, such as choosing jobs, renting offices, mutual funds investment, are demonstrated.

Solving the F-implicit generalized variational inequalities with applications

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Abstract. In this talk, we study the F-implicit generalized variational inequalities in a real normed space setting. Weak solutions and strong solutions are introduced. Several existence results are derived. As an application, we study the F-implicit generalized complementarity problems and some existence results are obtained.

0-1 Semidefinite Programming in Data Clustering: Modelling and Approximation

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Abstract. In this talk, we will introduce a novel optimization model called 0-1 semidefinite programming (0-1 SDP), which arises frequently in data cluster analysis. We show that various clustering models including the classical K-means clustering, the newly developed normalized cut problem and cluster ensemble models can be embedded into the 0-1 SDP model.

Next we discuss the solution techniques for the underlying 0-1 SDP problem. In particular, we focus on the development of approximation algorithms based on the relaxation of the 0-1 SDP model. Numerical experiments based on approximation algorithms will be reported.

A Dual Optimization Approach for Inverse Quadratic Eigenvalue Problems with Partial Eigenstructure

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Abstract. The inverse quadratic eigenvalue problem (IQEP) arises in the field of structural dynamics. It aims to find three symmetric matrices, known as the mass, the damping and the stiffness matrices, respectively such that they are closest to the given analytical matrices and satisfy the measured data. The difficulty of this problem lies in the fact that in applications the mass matrix should be positive definite and the stiffness matrix positive semidefinite. Based on an equivalent dual optimization version of the IQEP, we present a quadratically convergent Newton-type method. Our numerical experiments confirm the high efficiency of the proposed method.

Proximal Point Algorithms in Optimization and Four Nonlinear Projections

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Abstract. Let C be a nonempty closed convex subset of a real Hilbert space H and let $f : H \to (-\infty, \infty]$ be a proper convex lower semicontinuous function. Consider a convex minimization problem:

$$\min\{f(x) : x \in C\} = \alpha.$$

The number α is called an *optimal value*, C is called an *admissible set* and $M = \{y \in C : f(y) = \alpha\}$ is called an *optimal set*. Next, define a function $g : H \to (-\infty, \infty]$ as follows:

$$g(x) = \begin{cases} f(x), & x \in C, \\ \infty, & x \notin C. \end{cases}$$

Then, g is a proper lower semicontinuous convex function of H into $(-\infty, \infty]$. So, we consider the convex minimization problem:

$$\min\{g(x) : x \in H\}. \tag{$*$}$$

For such a g, we can define a multivalued operator ∂g on H by

$$\partial g(x) = \{x^* \in H : g(y) \geq g(x) + (x^*, y - x), y \in H\}$$

for all $x \in H$. Such a ∂g is said to be the *subdifferential* of g. An operator $A \subset H \times H$ is *accretive*, if for $(x_1, y_1), (x_2, y_2) \in A$,

$$(x_1 - x_2, y_1 - y_2) \geq 0.$$

If A is accretive, we can define, for each positive λ, the resolvent $J_\lambda : R(I + \lambda A) \to D(A)$ by $J_\lambda = (I + \lambda A)^{-1}$. We know that J_λ is a nonexpansive mapping. An accretive operator $A \subset H \times H$ is called *m-accretive*, if $R(I + \lambda A) = H$ for all $\lambda > 0$. If $g : H \to (-\infty, \infty]$ is a proper lower semicontinuous convex function, then ∂g is an m-accretive operator.

We know that one method for solving $(*)$ is the *proximal point algorithm* first introduced by Martinet. The proximal point algorithm is based on the notion of resolvent J_λ, i.e.,

$$J_\lambda x = \arg\min\Big\{g(z) + \frac{1}{2\lambda}\|z - x\|^2 : z \in H\Big\}.$$

The proximal point algorithm is an iterative procedure, which starts at a point $x_1 \in H$, and generates recursively a sequence $\{x_n\}$ of points $x_{n+1} = J_{\lambda_n} x_n$, where $\{\lambda_n\}$ is a sequence of positive numbers.

In this talk, we first prove weak and strong convergence theorems for resolvents of accretive operators and maximal monotone operators in Banach spaces. That is, we discuss weak and strong convergence of proximal point algorithms in Banach spaces with four nonlinear projections. One of four nonlinear projections is new.

Optimal State-Space Solution with Minimal Realization for Standard $\mathcal{H}_2$ Control Problem

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Abstract. In this talk, we will show that the controllable and unobservable subspaces of both the continuous and discrete-time $\mathcal{H}_2$ optimal controllers can be characterized by the image and kernel spaces of two matrices Z_2 and W_2, where Z_2 and W_2 are positive semidefinite solutions of two pertinent Lyapunov equations. The coefficients of the two Lyapunov equations involve the stabilizing solutions of the two celebrated Algebraic Riccati equations used in solving the $\mathcal{H}_2$ optimal control problem. By suitably choosing the bases adapted to ImZ_2 and KerW_2, the structure of the $\mathcal{H}_2$ optimal controller is geometrically clear. A minimal order state-space realization of $\mathcal{H}_2$ optimal controller is then given via an elegant geometric approach. In terms of the use of geometric language, all the results and proofs given are simple and clear.

Presenter: Po-Feng Wu (吳柏鋒).

Recent Progresses on Strong Convergence of the Proximal Point Algorithm

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Abstract. Rockafellar's proximal point algorithm (PPA) produces a sequence $\{x_n\}$ by $x_{n+1} = (I + c_n T)^{-1} x_n + e_n$, $n = 0, 1, \cdots$, where T is a maximal monotone operator in a Hilbert space H, $\{c_n\}$ is a sequence of parameters which is bounded below away from 0, and $\{e_n\}$ is the sequence of errors such that $\sum_n \|e_n\| < \infty$. It is well-known that $\{x_n\}$ is always weakly convergent to a zero of T (if any), but not always strongly convergent if dim $H = \infty$. In this talk, we will review some recent progresses on the strong convergence aspect of the PPA.