『最佳化理論及其應用』單日研討會

Optimization Day

國立台灣師範大學數學系

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Sponsored by

國家理論中心 (北區)
國科會數學推動中心
國立台灣師範大學數學系

Organized by

朱亮儒、陳界山

Schedule of Programs
Place : M106, Mathematics Building

March 2 (Thursday)	Speakers/Events
09:30 - 10:00	Registration
10:00 - 10:45	Jie Sun
10:45 - 11:00	*BREAK*
11:00 - 11:35	林來居 (Lai-Jiu Lin)
11:35 - 12:10	許瑞麟 (Ruey-Lin Sheu)
12:10 - 14:00	*LUNCH*
14:00 - 14:45	Paul Tseng
14:45 - 15:20	林智仁 (Chih-Jen Lin)
15:20 - 15:40	*BREAK*
15:40 - 16:15	賴漢卿 (Hang-Chin Lai)
16:15 - 16:50	陸 行 (Hsing Paul Luh)
16:50 - 17:25	吳憲忠 (Hsien-Chung Wu)
17:45	*DINNER*

The price of anarchy under nonlinear and asymmetric costs

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Abstract. We derive new bounds for the price of anarchy under nonlinear and asymmetric costs. The bounds depend on an additional factor called the intrinsic cost of the system and therefore tend to be more accurate than the current bounds that are dependant only on the degree of asymmetry of the Jacobian and the degree of the nonlinearity of the cost function.

Optimization Methods with Signal Denoising Applications

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Abstract. We consider an approach to signal denoising whereby a linear combination of wavelet bases is fitted to the noisy signal by maximizing the likelihood function minus an l_1-penalty on the coefficients of combination. The l_1-penalty induces sparsity of the coefficients and thus avoiding oversmoothing the fine scale features. For Gaussian noise, the maximization problem can be solved efficiently using a block coordinate minimization (BCM) method. For more general noise, the maximization problem can be solved using a primal-dual interior point method.

More generally, we consider the problem of minimizing the sum of a smooth function and a block-separable convex function. We propose a method based on applying BCM to a quadratic approximation of the smooth function at each iteration. This method is cheaper to implement, and has stronger convergence properties than BCM. In particular, it achieves linear convergence under a certain local error bound assumption. Numerical test results and comparison with MINOS 5.5.1 will be presented.

[The first part of the talk is joint work with Sylvain Sardy and Andrew Bruce. The second part is joint work with Sangwoon Yun.]

Systems of Generalized Quasi-Variational Inclusion Problems With Applications to Variational Analysis

林來居 (Lai-Jiu Lin)
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Abstract. In this talk, we study an existence theorem of systems of generalized quasi-variational inclusion problem. By this result, we establish the existence theorems of solutions of systems of generalized equations, systems of generalized quasiequilibrium problem, common fixed point theorem, systems of strong quasisaddle point, systems of weak quasisaddle point, systems of minimax theorem, mathematical program with systems of variational inclusion constraints, mathematical program with equilibrium constraints, systems of bilevel problem, semi-infinite problem with systems of equilibrium constraints and mathematical program with mix-variational inequalities constraints.

Projected Gradient Methods for Non-negative Matrix Factorization

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Abstract. Non-negative matrix factorization (NMF) is a useful unsupervised learning method. It requires the solution of a non-convex bound-constrained optimization problem. This work proposes two projected-gradient methods for NMF. Compared to existing approaches, the new methods have sound optimization properties. We discuss their efficient implementations and show that one proposed method converges faster than the popular multiplicative update approach.

The Karush-Kuhn-Tucker Optimality Conditions for the Optimization Problem with Fuzzy-Valued Objective Function

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Abstract. The KKT conditions for an optimization problem with fuzzy-valued objective function will be derived in this talk. A solution concept of this optimization problem is proposed by considering an ordering relation on the class of all fuzzy numbers. The proposed solution concept will follow from the similar solution concept, called nondominated solution, in the multiobjective programming problem. In order to consider the differentiation of an fuzzy-valued function, we invoke the Hausdorff metric to define the distance between two fuzzy numbers and the Hukuhara difference to define the difference of two fuzzy numbers. Under these settings, the KKT optimality conditions will be elicited naturally by introducing the Lagrange function multipliers.

An application in comparison of queues by stochastic directional convexity

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Abstract. Second order properties of queues are important in design and analysis of service systems. In this talk we show that the blocking probability of $M/M/C/N$ queue is increasing directionally convex in $(\lambda, -\mu)$, where λ is arrival rate and μ is service rate. Consider a heterogeneous queueing system with non-stationary arrival and service processes. The arrival and service rates alternate between two levels (λ_1, μ_1) and (λ_2, μ_2), spending an exponentially distributed amount of time with rate $c\alpha_i$ in level i, $i = 1, 2$. When the system is in state i, the arrival rate is λ_i and the service rate is μ_i. Applying the increasing directional convexity result we show that the blocking probability is decreasing in c. In fact, we show that a stronger result holds: the number of blocked customers by time t is decreasing in c in stochastic convex ordering for any $t \geq 0$. In particular, when $N = \infty$, the result is reduced to that the number of customers in system at any time t is decreasing in c in stochastic convex ordering.

Modified Dinkelbach-Type Algorithm for Generalized Fractional Programs with Infinitely Many Ratios

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Abstract. We extend the Dinkelbach-type algorithm of Crouzeix, Ferland, and Schaible to solve min-max fractional programs with infinitely many ratios. Parallel to the case with finitely many ratios, the task is to solve a sequence of continuous min-max problems,

$$P(\alpha_k) = \min_{x \in X} \left(\max_{t \in T} [f_t(x) - \alpha_k g_t(x)] \right),$$

until $\{\alpha_k\}$ converges to the root of $P(\alpha) = 0$. The solution of $P(\alpha_k)$ is used to generate α_{k+1}. However, calculating the exact optimal solution of $P(\alpha_k)$ requires an extraordinary amount of work. To improve, we apply an entropic regularization method which allows us to solve each problem $P(\alpha_k)$ incompletely, generating an approximate sequence $\{\tilde{\alpha}_k\}$, while retaining the linear convergence rate under mild assumptions. We present also numerical test results on the algorithm which indicate that the new algorithm is robust and promising.

Optimality Conditions for Integral Functional Involving Control Problem

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Abstract. The classical variational problem is to minimize the integral functional $J : X \longrightarrow \mathbf{R}$ as the form:

$$J(x) = \int_a^b g(t, x(t), \dot{x}(t))dt$$

subject to $x \in X = C^1(T, \mathbf{R^n})$, $x(a) = \alpha, x(b) = \beta$ in $\mathbf{R^n}$, where the integrand $g(t, x(t), \dot{x}(t)) : T \times \mathbf{R}^n \times \mathbf{R^n} \longrightarrow \mathbf{R}$ is continuous in $t \in T = [a, b]$ and having continuous partial derivative w.r.t. x and $\dot{x}, x \in C^1(T, \mathbf{R^n})$. It is known that the optimal solution x satisfies the Euler- Lagrange eq.

$$\frac{d}{dt} g_{\dot{x}}(x, \dot{x})(t) - g_x(x, \dot{x})(t) = 0.$$

This result is extended to non-convex, non-locally Lipschitz integrand involving control problem as the form:

$$\begin{array}{lll} (P_c) & \text{Min} & J(x,u) = \int_a^b g(t, x(t).\dot{x}(t), u(t))\ dt \\ & \text{such that} & \dot{x} \in F(t, x(t), u(t)), u(t) \in U(t) \subset Y \end{array}$$

which is reduced to an unconstrained problem:

$$\begin{array}{lll} (P) & \text{Min} & J(x) = F(x, \dot{x}) = \int_a^b L_t(x, \dot{x})\ dt \\ & \text{such that} & x \in A_X^p, 1 \leq p \leq \infty \end{array}$$

where $L_t(x, \dot{x}) = \inf u(t) \in U(t) g(t, x(t), \dot{x}(t), u(t))$, is non convex, non locally Lipschitz; $A_X^p = \{x \in A^p(T, X) | \dot{x} \in L^p(T, X)\}$ with $|||x||| = \|x(t)\|_X + \|\dot{x}\|_p, 1 \leq p \leq \infty$; $F : X \times X \longrightarrow 2^X$ is a multi-map and $g : T \times X \times X \times Y \longrightarrow (-\infty, +\infty]$ is integrable in $t \in T$; and X and Y are separable Banach spaces.

In this talk, we will find the optimality (necessary) condition which satisfies the generalized Euler- Lagrange Equation, that is, there exists an absolutely continuous function $\alpha \in A_X^1$ such that

$$(\dot{\alpha}, \alpha(t)) \in \partial^o L_t(z, \dot{z})$$

for almost all $t \in T$ where $\partial^o L_t(z, \dot{z})$ is Clarke subgradient differential provided that z is an optimal solution of (P) and $L_t(z, \dot{z})$ is measurable in t satisfying the quasi locally Lipschitz at $(z, \dot{z})$. If $L_t(\cdot, \cdot)$ is continuous differentiable $w.r.t.$ $(s, v) \in X \times X$, then

$$\frac{d}{dt} L_{t,\dot{x}}(z, \dot{z}) = L_(t, x)(z, \dot{z}).$$