Workshop on Nonlinear Analysis and Optimization

Department of Mathematics
National Taiwan Normal University

December 17-19, 2008

Sponsored by

National Science Council
National Taiwan Normal University

Organized by

Jein-Shan Chen, Mau-Hsiang Shih

Schedule of Programs
Place : M210, Mathematics Building

Table 1: December 17, Wednesday

	Chair	Speaker	Title
09:30 - 10:15	M-H Shih	Wataru Takahashi	Nonlinear operators in optimization and nonlinear analysis
		Tea Break	
10:25 - 11:10	M-H Shih	Hang-Chin Lai	Approximate solutions and error bounds for an integral functional programming problem
		Tea Break	
11:20 - 12:05	M-H Shih	Shigeo Akashi	Hilbert's 13th problem and the multidimensional numerical data compression
		Lunch	
13:30 - 14:15	D.S. Kim	Lai-Jiu Lin	Systems of set-valued vectorial version of Ekeland's variational principle
		Tea Break	
14:25 - 15:10	S. Schaible	Ruey-Lin Sheu	Global optimization for a class of fractional programming problems
		Tea Break	
15:20 - 16:05	M-S Shih	Feng-Sheng Tsai	Information storage scheme in associative memory
		Tea Break	
16:15 - 17:00	L-J Lin	Jong-Kyu Kim	A new system of generalized mixed quasi-variational inclusions involving (A, η)- accretive mappings

Table 2: December 18, Thursday

	Chair	Speaker	Title
09:30 - 10:15	T. Tanaka	Do Sang Kim	Duality and stability for a class of nonconvex programs with an infinite number of constraints
		Tea Break	
10:25 - 11:10	J-S Chen	Chein-Shan Liu	Novel methods to solve nonlinear algebraic equations and optimization problems
		Tea Break	
11:20 - 12:05	J-S Chen	Hong-Kun Xu	Strong convergence for the proximal point algorithm in nonsmooth Banach spaces
		Lunch	
13:30 - 14:15	H-C Lai	Shue-Chin Huang	Hybrid proximal point algorithms for solving constrained minimization problems in Banach spaces
		Tea Break	
14:25 - 15:10	H-C Lai	Siegfried Schaible	The abstract equilibrium problem
		Tea Break	
15:20 - 16:05	H-C Lai	Tamaki Tanaka	A unified approach for scalarization on sets

Table 3: December 19, Friday

	Chair	Speaker	Title
09:30 - 10:10	F-S Tsai	Ching-Yu Yang	Recurrent neural networks for solving second-order cone programs
		Tea Break	
10:20 - 11:00	F-S Tsai	Shen-Yi Hsu	Complexity of economics
		Tea Break	
11:10 - 11:50	F-S Tsai	Shu-Han Wu	The Szemerédi's theorem on arithmetic progressions
		Lunch	

Hilbert's 13th problem and the multidimensional numerical data compression

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Abstract. In 1957, Kolmogorov and Arnold proved affirmatively the 13th problem which Hilbert had formulated in 1900. Actually, it is known that there still exist various kinds of unsolved problems which have been derived from Hilbert's 13th problem. In this talk, the negative solution to the 13th problem in case of infinitely differentiable functions of several variables is given, and this result is applied to the problem asking what extent the multidimensional numerical data can be compressed to.

Complexity of Economics

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Abstract. This talk will present some dynamical aspects of the Arrow-Debreu general economic equilibrium model and explain why we can regard the equilibrium as an emergent phenomenon.

Hybrid Proximal Point Algorithms for Solving Constrained Minimization Problems in Banach Spaces

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Abstract. The purpose of this talk is to analyze new hybrid proximal point algorithms and solve the constrained minimization problem involving a convex functional in a uniformly convex and uniformly smooth Banach space. The results presented in this talk improve and develop some results in the recent literature.

Co-workers: L. C. Ceng and Y. C. Liou.

Duality and Stability for a Class of Nonconvex Programs with an Infinite Number of Constraints

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Abstract. In this talk, we introduce a class of nonconvex programming problems which has an infinite number of constraints. Duality relations for nonconvex programs corresponding to some kinds of dual problems are established by using the property of semiconvexity. Moreover, we extend the stability and saddle point theorems for convex programming to nonconvex infinite problems.

2000 *Mathematics Subject Classification.* 90C26, 90C34, 49N15, 90C46.

Key words and phrases. Semiconvexity, Nonconvex infinite program, Duality, Saddle point, Stability, Augmented Lagrangian.

A new system of generalized mixed quasi-variational inclusions involving (A, η)- accretive mappings

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Abstract. In this talk, a new system of generalized mixed quasi-variational inclusions involving (A, η)-accretive mappings in Banach spaces is introduced and studied, which includes many kind of variational inequality (inclusion) problems as special cases. Using the resolvent operator methods associated with (A, η)-accretive mappings, an existence and uniqueness theorem of solutions for this system. Some new iterative algorithms for finding approximate solutions of this system is suggested and discussed, the convergence theorem of iterative sequence generated by new iterative algorithms is also given.

Approximate solutions and error bounds for an integral functional programming problem

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Abstract. Consider an integral functional programming as

$$\begin{aligned} \text{(P)} \quad \max \quad & \int_0^T f(t)^T x(t)dt \quad (= \langle x, f \rangle) \\ \text{s.t.} \quad & x \in L^\infty([0,T], \mathbb{R}^q_+) \quad \text{and} \quad \text{for} \quad t \in [0,T], \\ & B(t)x(t) - \int_0^t K(s,t)x(s)ds \leq_{\mathbb{R}^p_+} g(t) \quad \text{in} \quad \mathbb{R}^p \end{aligned}$$

where $B(t)$ and $K(s,t)$ are $p \times q$ matrices; $f(\cdot) \in C([0,T], \mathbb{R}^q_+)$, $g(\cdot) \in C([0,T], \mathbb{R}^p_+)$; and ^{T} means transpose operation.

Purpose of this talk include

1. to find a sequence of approximate solutions to be convergent in L^∞ and
2. to establish a formula of error bounds for the approximate solutions,
3. to evaluate the results by two examples and give the practical approximate solutions.

Systems of Set-Valued Vectorial Version of Ekeland's Variational Principle

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Abstract. In this paper, we apply an abstract maximal element theorem to study various types of set-valued vectorial version of Ekeland's variational principle for maps defined on metric space or product of metric spaces with quasiorder complete. Our results extend and improve many results on Ekeland's variational principle and Caristi's fixed point in the literature.

Novel Methods to Solve Nonlinear Algebraic Equations and Optimization Problems

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Abstract. In this talk I report a novel technique to solve nonlinear algebraic equations (NAEs) basing on a new concept of fictitious time integration method (FTIM), which transforms the NAEs into a mathematical equivalent system of ordinary differential equations. Then I report a novel technique to solve the nonlinear optimization problem (NOP) under multiple equality and inequality constraints. The Kuhn-Tucker optimality conditions are used to transform the NOP into a mixed complementarity problem (MCP). With the aid of NCP-functions a set of nonlinear algebraic equations is obtained. Then we can apply the FTIM to solve these NAEs, and thus the NOP is solved.

The Abstract Equilibrium Problem

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Abstract. A unifying model of optimization problems and classical problems in nonlinear analysis is the abstract equilibrium problem. We present eight realizations of this rather general model and point out some applications through these special cases. The classical assumption of convexity and monotonicity can be relaxed to generalized convexity and generalized monotonicity to establish major results.

Global Optimization for a Class of Fractional Programming Problems

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Abstract. This talk presents a canonical dual approach to the problem of minimizing the sum of a quadratic function and the ratio of two quadratic functions, which is a type of non-convex optimization problem subject to an elliptic constraint. We first relax the fractional structure by introducing a family of parametric subproblems. Under proper conditions on the "problem-defining" matrices associated with the three quadratic functions, we show that the canonical dual of each subproblem becomes a one-dimensional concave maximization problem that exhibits no duality gap. Since the infimum of the optima of the parameterized subproblems leads to a solution to the original problem, we then derive some optimality conditions and existence conditions for finding a global minimizer of the original problem. Some numerical results using quasi-Newton and line search methods are presented to illustrate our approach.

Nonlinear Operators in Optimization and Nonlinear Analysis

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Abstract. Let E be a real Banach space and let E^* be the dual space of E. Let C be a closed convex subset of E. Let f be a bifunction from $C \times C$ to R, where R is the set of real numbers. The equilibrium problem is to find $\hat{x} \in C$ such that

$$f(\hat{x}, y) \geq 0, \quad \forall y \in C. \tag{1}$$

The set of solutions of (1) is denoted by $EP(f)$. Numerous problems in physics, optimization, and economics reduce to find a solution of (1).

In this talk, motivated by the equilibrium problem, we first introduce some nonlinear operators in Banach spaces. Next, we prove fundamental results for the nonlinear operators which are related to Optimization and Nonlinear Analysis. Finally, we apply these results to prove some important results in Banach spaces or Hilbert spaces.

A Unified Approach for Scalarization on Sets

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Abstract. Hamel and Löhne [2] proved the existence results for minimal points of subsets of the product space $X \times 2^Y$ by using the result of [1], where X and Y are a separated uniform space and a topological vector space, respectively. In this paper, we investigate several kinds of scalarization functionals on sets and present a minimal element theorem with set-relations, which is similar to one of those in [2] but proved by using a different scalarization functional (see [4]).

Let Y be a real ordered topological vector space with the vector ordering $\leq_C$ induced by a convex cone $C \subset Y$; for $x, y \in Y$, $x \leq_C y$ if $y - x \in C$. We define the following sublinear scalarization function:

$$h_C(y; k) := \inf\{t \,|\, y \in tk - C\} \tag{2}$$

where $k \in \operatorname{int} C$. We get $-h_C(-y; k) = \sup\{t \,|\, y \in tk + C\}$. Based on these ideas and several scalarization functions introduced in [2], we define the following 12 types of scalarization functions on 2^Y with respect to direction $k \in C$ and reference set $V' \subset Y$ by using $\leq_C^{(i)}$ $(i = 1, \ldots, 6)$ in [3]; they are unified forms including all scalarization functions introduced in [2].

$$\mathrm{I}^{(j)}_{k,V'}(V) := \inf\left\{t \in \mathbb{R}\right\} \Big| V \leq_C^{(j)} (tk + V')\Big\}, \quad (j = 1, \ldots, 6) \tag{3}$$

$$\mathrm{S}^{(j)}_{k,V'}(V) := \sup\left\{t \in \mathbb{R}\right\} \Big| (tk + V') \leq_C^{(j)} V\Big\}, \quad (j = 1, \ldots, 6) \tag{4}$$

where V is a subset of Y and the set-relationships in [3] are defined as follows:

(i) $A \leq_C^{(1)} B$ by $A \subset \cap_{b\in B}(b - C)$, (ii) $A \leq_C^{(2)} B$ by $A \cap (\cap_{b\in B}(b - C)) \neq \phi$,
(iii) $A \leq_C^{(3)} B$ by $\cup_{a\in A}(a + C) \supset B$, (iv) $A \leq_C^{(4)} B$ by $(\cap_{a\in A}(a + C)) \cap B \neq \phi$,
(v) $A \leq_C^{(5)} B$ by $A \subset \cup_{b\in B}(b - C)$, (vi) $A \leq_C^{(6)} B$ by $A \cap (\cup_{b\in B}(b - C)) \neq \phi$.

Topics: Set-Valued Analysis, Nonlinear Scalarization

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Co-workers: Akira Shimizu, Syuuji Yamada.

Information Storage Scheme in Associative Memory

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Abstract. A major puzzle in neural networks is understanding the information encoding principles that implement the functions of the brain systems. Population coding in neurons and plastic changes in synapses are two important subjects in attempts to explore such principles. This forms the basis of modern theory of neuroscience concerning self-organization and associative memory. Here we wish to suggest an information storage scheme based on the dynamics of evolutionary neural networks, essentially reflecting the meta-complication of the dynamical changes of neurons as well as plastic changes of synapses. The information storage scheme may lead to the development of a complete description of all the equilibrium states of Hopfield networks, a space-filling network that weaves the intricate structure of Hamming star-convexity, and a plasticity regime that encodes information based on algorithmic Hebbian synaptic plasticity.

The Szemerédi's Theorem on Arithmetic Progressions

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Abstract. In 1936 Erdös and Turan conjectured that for every value d called density $0 < d < 1$ and every integer k there is a number $N(d, k)$ such that every subset A of $\{1, ..., N\}$ of cardinality dN contains a length-k arithmetic progression, provided $N > N(d, k)$. In 1975 Endre Szemerédi proved this conjecture by a combinatorial method. In this talk, we will present a dynamical proof obtained by Harry Furstenberg of the Szemerédi's theorem on arithmetic progressions.

Strong Convergence for the Proximal Point Algorithm in Nonsmooth Banach Spaces

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Abstract. We provide a strongly convergent modification of Rockafellar's proximal point algorithm in a Banach space which is uniformly convex with convexity of power two and which is not necessarily smooth.

Recurrent Neural Networks for Solving Second-Order Cone Programs

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Abstract. This paper proposes using the neural networks to efficiently solve the second-order cone programs (SOCP). To establish the neural networks, the SOCP is first reformulated as a second-order cone complementarity problem (SOCCP) with the Karush-Kuhn-Tucker conditions of the SOCP. The SOCCP functions, which transform the SOCCP into a set of nonlinear equations, are then utilized to design the neural networks. We propose two kinds of neural networks with the different SOCCP functions. The first neural network uses the Fischer-Burmeister function to achieve an unconstrained minimization with a merit function. We show that the merit function is a Lyapunov function and this neural network is asymptotically stable. The second neural network utilizes the natural residual function with the cone projection function to achieve low computation complexity. It is shown to be Lyapunov stable and converges globally to an optimal solution under some condition. The SOCP simulation results demonstrate the effectiveness of the proposed neural networks.

Co-workers: Chun-Hsu Ko, Jein-Shan Chen.