Optimization Meeting

Department of Mathematics National Taiwan Normal University

November 12, 2008

Sponsored by

Division of Mathematics, National Center for Theoretical Sciences, Taipei Office Mathematics Research Promotion Center, NSC Office of International Affairs, National Taiwan Normal University

Organized by

Jein-Shan Chen

Schedule of Programs Place : M210, Mathematics Building

November 12	Speakers/Events	Titles of Talks
09:10 - 09:50	Hsing Luh	Constructing an efficient search direction for linear
		programming problems
09:50 - 10:30	Shaohua Pan	A proximal gradient method for the extended
		second-order cone linear complementarity problem
10:30 - 10:50	BREAK	
10:50 - 11:30	Naihua Xiu	Löwner Operators in Euclidean Jordan Algebras
11:30 - 12:10	Shuechin Huang	Existence Theorems for Generalized Vector Variational
		Inequalities with a Variable Ordering Relation
12:10 - 14:10	LUNCH	
14:10 - 14:50	Vincent F. Yu	A Simulated Annealing Heuristic for the Truck and
		Trailer Routing Problem with Time Windows
14:50 - 15:30	Chun-Nan Hsu	Learning from Infinite Many Training Examples
15:30 - 15:50	BREAK	
15:50 - 16:30	Jen-Yen Lin	Continuous Min-Max Programming with Semi-infinite
		Constraints
16:30 - 17:10	Shyan-Shiou Chen	The Chaotic and Convergent Dynamics in Neural
		Networks

The Chaotic and Convergent Dynamics in Neural Networks

Shyan-Shiou Chen Department of Mathematics National Taiwan Normal University Taipei 11677, Taiwan E-mail: sschen@ntnu.edu.tw

Abstract. Recently, chaotic neural networks have been paid much attention to, and contribute toward solving TSP. We study the existence of chaos in a discrete-time neural network. Chaotic behavior is an inside essence of stochastic processes in nonlinear deterministic system. The investigation provides a theoretical confirmation on the scenario of transient chaos for the system. All the parameter conditions for the theory can be examined numerically. The numerical ranges for the parameters which yield chaotic dynamics and convergent dynamics provide significant information in the annealing process in solving combinatorial optimization problems using this transiently chaotic neural network.

Learning from Infinite Many Training Examples

Chun-Nan Hsu Institute of Information Science Academia Sinica Taipei 115, Taiwan E-mail: chunnan@iis.sinica.edu.tw

Abstract. Previously, it has been established that the second-order stochastic gradient descent (2SGD) method can potentially achieve generalization performance as well as empirical optimum in a single pass through the training examples. However, 2SGD requires computing the inverse of the Hessian matrix of the loss function, which is prohibitively expensive. This talk presents "Componentwise Triple Jump" (CTJ) and "Periodic Step-size Adaptation" (PSA), which approximates the Jacobian matrix of the mapping function and explores a linear relation between the Jacobian and Hessian to approximate the Hessian periodically and achieve near-optimal results in experiments on a wide variety of models and tasks. With a single-pass method, it becomes practical for a computer to learn from a stream of infinite many training examples as humans do.

Existence Theorems for Generalized Vector Variational Inequalities with a Variable Ordering Relation

Shuechin Huang Department of Applied Mathematics National Dong-Hwa University Hualien 97401, Taiwan E-mail: shuang@mail.ndhu.edu.tw

Abstract. We study the solvability of the generalized vector variational inequality problem, the GVVI problem, with a variable ordering relation in reflexive Banach spaces. The existence results of strong solutions of GVVIs for monotone multifunctions are established with the use of Fan-KKM Theorem. We also investigate the GVVI problems without monotonicity assumptions and obtain the corresponding results of weak solutions by applying Brouwer fixed point theorem. These results are also the extension and improvement of some recent results in the literature.

Continuous Min-Max Programming with Semi-infinite Constraints

Jen-Yen Lin Department of Applied Mathematics National Chiayi University Chia-Yi, Taiwan E-mail: jylin@mail.ncyu.edu.tw

Abstract. In this paper, we propose an algorithm for solving a kind of nonlinear programming where the objective is the maximal function of a family of continuous functions and the feasible domain is explicitly made of infinitely many constraints. Our algorithm combines the entropic regularization and the cutting plane method (the Remez-type) to deal with the non-differentiability of the maximal function and the infinitely many constraints respectively. A finite inexact version, which terminates within a finite number of iterations to give an approximate solution, is proposed to handle the computational issues, including the blow-up problem in the entropic regularization and the global optimization subproblems in the cutting plane method. To justify the efficiency of the inexact algorithm, we also analyze the theoretical error-bound and conduct numerical experiments.

Constructing an efficient search direction for linear programming problems

Hsing Luh Department of Mathematical Sciences National Cheng-Chi University Taipei 11605, Taiwan E-mail: slu@nccu.edu.tw

Abstract. In this talk, we present an auxiliary algorithm, in terms of the speed of obtaining the optimal solution, that is efficient in helping the simplex method for commencing a better initial basic feasible solution. The idea of choosing a direction towards an optimal point is easy to implement. From our experiments, the algorithm will release a corner point of the feasible region within few iterative steps, independent of the starting point. The computational results show that after the auxiliary algorithm is adopted as phase I process, the simplex method consistently reduce the number of required iterations by about 40

A proximal gradient method for the extended second-order cone linear complementarity problem

Shaohua Pan School of Mathematical Sciences South China University of Technology Guangzhou 510640, China E-mail: shhpan@scut.edu.cn

Abstract. We consider an extended second-order cone linear complementarity problem (SOCLCP), which includes as special cases the generalized SOCLCP, the horizontal SOCLCP, the vertical SOCLCP, and the mixed SOCLCP. In this paper, we present some simple second-order cone constrained reformulation problems and unconstrained reformulation problems, and under mild conditions prove the equivalence between the KKT points of the optimization problems and the solutions of the extended SOCLCP. Particularly, we propose a proximal gradient descent method for solving the equivalent second-order cone constrained problems. This method is very simple and makes only one Euclidean projection onto second-order cones at each iteration. We establish global convergence and, under a local Lipschitzian error bound assumption, local linear rate of convergence for this method. Numerical results are reported, and made comparisons with those given by the limited BFGS method for solving the unconstrained reformulation problems, which verify the effectiveness of the proposed method.

Löwner Operators in Euclidean Jordan Algebras¹

Naihua Xiu Department of Applied Mathematics Beijing Jiaotong University Beijing 100044, P.R. China E-mail: nhxiu@center.njtu.edu.cn

Abstract. In 1934, the famous mathematician Karl Löwner defined a matrix function which was called Löwner operator by Sun and Sun (2008): Consider a real-valued scalar function $g: (a, b) \to \mathbb{R}$. Such a function can be used to define an analogous operator G on the *n*-by-*n* symmetric matrices over the reals. That is, if x has the spectral decomposition

$$x = \sum_{i=1}^{n} \lambda_i(x) u_i u_i^T$$

then

$$G(x) := \sum_{i=1}^{n} g\left(\lambda_i(x)\right) u_i u_i^T,$$

where $\lambda_i(x)$ and u_i $(i = 1, 2, \dots, n)$ are the eigenvalues and the corresponding eigenvectors of x, respectively. The domain of g implies a corresponding domain for G.

Löwner operator has special structure and properties, and has important applications in electrical networks, elementary particles, statistical analysis, etc.. It becomes one of the main contents in monograph or text "Matrix Analysis".

This talk focuses on Löwner operator in Euclidean Jordan algebras and its applications to the symmetric cone optimization problems and others, which is based on speaker's recent joint work, and also the work by Korányi (1984), Sun and Sun (2008), and Baes (2007). The contents are as follows.

- Definition of Löwner Operator
- Differentiability of Löwner Operator
- Semismoothness of Löwner Operator
- Monotonicity of Löwner Operator

¹The work was partly supported by the National Natural Science Foundation of China (10671010, 70640420143).

- Operator-Monotonicity of Löwner Operator
- Applications of Löwner Operator
- Some Problems

A Simulated Annealing Heuristic for the Truck and Trailer Routing Problem with Time Windows

Vincent F. Yu Department of Industrial Management National Taiwan University of Science and Technology Taipei 106, Taiwan E-mail: vincent@mail.ntust.edu.tw

Abstract. In this study, we consider the application of a simulated annealing (SA) heuristic to the truck and trailer routing problem with time windows (TTRPTW), an extension of the truck and trailer routing problem (TTRP). TTRP is a variant of the well-known well-studied vehicle routing problem (VRP). In TTRP, some customers can be serviced by either a complete vehicle (that is, a truck pulling a trailer) or a single truck, while others can only be serviced by a single truck for various reasons. In some VRP applications, each customer has a predetermined time window for accepting services. This problem is known as the vehicle routing problem with time windows (VRPTW), and belongs to the class of NP-hard problems. Similarly, the time window constraints may be imposed on TTRP applications, and the resulting problem is called the truck and trailer routing problem with time windows. It can be easily verified that VRPTW is a special case of TTRPTW. Thus TTRPTW also belongs to the class of NP-hard problems and it is natural to tackle this problem with heuristics approaches.

Simulated annealing (SA) has seen widespread applications to various combinatorial optimization problems, including the VRPTW and TTRP which are closely related to the TTRPTW. Therefore, we developed an SA based heuristic to solve TTRPTW. The proposed SA heuristic was first tested on six Solomon's VRPTW benchmark problems to validate its effectiveness in solving VRPTW type of problems. To our best knowledge, there are no benchmark instances for TTRPTW in the literature. Therefore, we converted 12 Solomon's VRPTW benchmark problems and six Homberger's extended Solomon's VRPTW instances into 54 TTRPTW benchmark problems and tested our SA heuristic on them. Computational study indicates that SA is capable of consistently producing high quality solutions to TTRPTW within a reasonable time.

Co-authors: Shih-Wei Lin, Chung-Cheng Lu.

Keywords: simulated annealing, vehicle routing problem, truck and trailer routing problem, time window.