2009 Workshop on Nonlinear Analysis and Optimization

Department of Mathematics
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Organized by

Jein-Shan Chen, Mau-Hsiang Shih

Schedule of Programs
Place : M210, Mathematics Building

Table 1: November 25, Thursday

	Chair	Speaker	Title
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10:25 11:10	M-H Shih	S. Dhompongsa	Fixed points of multivalued mappings in Banach and metric spaces
11:20 12:05	M-H Shih	L-J Lin	Generalized Variational Relation Problems With Applications
		Lunch Break	
13:30 14:15	S. Akashi	H-K Xu	Splitting Methods for Monotone Operators and Their Applications in Signal Recovery
14:25 15:10	S. Akashi	T. Ibaraki	Fixed point theorems for nonlinear mappings of nonexpansive type in Banach spaces
15:20 16:05	L-J Lin	C-T Pong	Stability Analysis of Interval Systems
16:15 17:00	L-J Lin	S. Akashi	Hilbert's 13th problem and nonlinear approximation problems

Table 2: November 26, Thursday

	Chair	Speaker	Title
09:30 10:15	J-S Chen	L-Q Qi	Higher Order Positive Semi-Definite Diffusion Tensor Imaging and Space Tensor Conic Programming
10:25 11:10	J-S Chen	S-M Guu	On a recurrence algorithm for the continuous-time linear fractional programming problems
11:20 12:05	J-S Chen	S. Yun	An accelerated proximal gradient algorithm for nuclear norm regularized least squares problems
		Lunch Break	
13:30 14:15	H-K Xu	J. K. Kim	Strong convergence of hybrid projection methods for fixed point and equilibrium problems
14:25 15:10	H-K Xu	N-C Wong	An unified hybrid iterative method for zeros or fixed points of nonlinear operators
15:20 16:05	N-C Wong	Y. Kimura	Shrinking projection methods for families of nonlinear mappings
16:15 17:00	N-C Wong	D. S. Kim	Duality and Sufficiency in Nonsmooth Multiobjective Programming

Table 3: November 27, Friday

	Chair	Speaker	Title
09:10 09:50	J-S Chen	Y-L Chang	Strong semismoothness of Fischer-Burmeister complementarity function associated with symmetric cone
09:55 10:35	J-S Chen	T. Honda	Nonlinear retractions in Banach spaces
10:40 11:20	J-S Chen	J-H Sun	Numerical Methods for solving cone constrained variational inequality problems
11:25 12:05	J-S Chen	F-S Tsai	Collective Dynamics Decoding with Neural Circuit Labeling

Hilbert's 13th problem and nonlinear approximation problems

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Abstract. Hilbert's 13th problem is famous as the problem asking whether or not any continuous functions of three variables can be represented as a superposition constructed from several continuous functions of two variables. Though Kolmogorov and Arnold solved this problem affirmatively, it is known that there remain some open problems which have been derived from the original problem. In this talk, it is shown that these derivative problems can be classified into several cases and that this result can be applied to nonlinear approximation problems.

Strong semismoothness of Fischer-Burmeister complementarity function associated with symmetric cone

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Abstract. We provide an affirmative answer to an open question that the Fischer-Burmeister complementarity function associated with symmetric cones, named the FB SC complementarity function, is globally Lipschitz continuous and strongly semismooth everywhere. This is achieved with the help of an operator from a finite dimensional vector space to its linear symmetric operator space.

Fixed points of multivalued mappings in Banach and metric spaces

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Abstract. In this talk, we will recall some geometric properties on Banach spaces recently developed. Then we demonstrate how our techniques can be used to solve some open problems. In the second part of the talk, we will present some new results in the metric space setting concerning selections, Schauder fixed point theorem and Kakutani fixed point theorem.

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On a recurrence algorithm for the continuous-time linear fractional programming problems

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Abstract. In this talk, we develop a discrete approximation method for solving continuous-time linear fractional programming problems. Our method enables one to derive a recurrence structure which shall overcome the computational curse caused by the increasing number of decision variables in the approximate problems when the subintervals are getting smaller and smaller. Convergence of our method will be proven and estimation for the error bounds of the approximate solutions will be provided.

Nonlinear retractions in Banach spaces

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Abstract. Recently, four nonlinear retractions (projections) were found in a Banach space. They are the metric projection, a sunny nonexpansive retraction, the generalized projection and a sunny generalized nonexpansive retraction. In a Hilbert space, they are equivalent to the metric projection. They can also related to the resolvents of accretive operators or monotone operators. Using these retractions, we can solve variational inequalities and fixed point problems in a Banach space.

In this talk, we consider one of them, a sunny generalized nonexpansive retraction. It was found by Ibaraki and Takahashi few years ago. Using this retraction, we obtain the generalization of the orthogonal complemented subspace decomposition in a Hilbert space. From this, we obtain the relation between contractive linear projections and sunny generalized nonexpansive retractions. Further, we discuss the equivalent conditions for a closed half space in a a Banach space to be a nonexpansive retract.

Keywords: Metric projection, generalized projection, nonexpansive retraction.

Subject classification: Primary 47H09; Secondary 60G05.

Fixed point theorems for nonlinear mappings of nonexpansive type in Banach spaces

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Abstract. In this talk, we introduce two nonlinear mappings of nonexpansive type which are connected with resolvents of maximal monotone operators in a Banach space. We first study some properties of these mappings. Next, we prove fixed point theorems and convergence theorems for these mappings.

Duality and Sufficiency in Nonsmooth Multiobjective Programming

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Abstract. In this talk, we introduce the nonsmooth multiobjective programming problem involving locally Lipschitz functions and support functions. We propose two types of Karush-Kuhn-Tucker conditions which differ only in the nonnegativity of the multipliers for the equality constraints. Sufficient Karush-Kuhn-Tucker optimality conditions are presented by using generalized convexity assumptions and certain regularity conditions. In addition, we formulate the Wolfe type dual and Mond-Weir type dual problems and establish duality relations for Pareto-optimal and weak Pareto-optimal solutions under suitable generalized convexity and regularity conditions.

Strong convergence of hybrid projection methods for fixed point and equilibrium problems

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Abstract. In this talk, we consider a hybrid projection method for finding a common element in the fixed point set of an asymptotically quasi-ϕ-nonexpansive mapping and in the solution set of an equilibrium problem. Strong convergence theorems of common elements are established in a uniformly smooth and strictly convex Banach space which also enjoys the Kadec-Klee property.

Keywords: Asymptotically quasi-ϕ-nonexpansive mapping; relatively nonexpansive mapping; generalized projection; equilibrium problem; lower semi-continuous

AMS Subject Classification: 47H09, 47J25

Shrinking projection methods for families of nonlinear mappings

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Abstract. Approximating solutions of the common fixed point problem has been studied by many researchers for various types of nonlinear mappings; contractions, nonexpansive mappings, relatively nonexpansive mappings, and others. In 2008, Takahashi, Takeuchi, and Kubota established strong convergence of an iterative scheme with a new type of hybrid method, also known as the shrinking projection method, for a family of nonexpansive mappings.

Motivated by this result, many authors have investigated this method and applied it to other types of nonlinear mappings. In this talk, we show these results and some recent development related to this result. The method of the proofs is different from the original one.

Generalized Variational Relation Problems With Applications

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Abstract. In this paper, we first obtain an existence theorem of the solutions of a variational relation problem. An existence theorem for a variational inclusion problem, a KKM theorem and several equilibrium theorems will be established as particular cases. Some applications concerning a saddle point problem with constraints, existence of a common fixed point for two mappings and an optimization problem with constraints, will be given in the last section of the paper.

Stability Analysis of Interval Systems

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Abstract. Interval matrix structures are ubiquitous in nature and engineering. Ordinarily, in an uncertain system there is associated with a set of coupled interval matrices, a basic issue of exploring its asymptotic stability. Here we introduce the notion of simultaneous Schur stability by linking the concepts of the majorant and the joint spectral radius, and prove the asymptotic stability of a set of interval matrices governed by simultaneous Schur stability. The present result may lead to the stability analysis of discrete dynamical interval systems.

Higher Order Positive Semi-Definite Diffusion Tensor Imaging and Space Tensor Conic Programming

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Abstract. Due to the well-known limitations of diffusion tensor imaging (DTI), high angular resolution diffusion imaging (HARDI) is used to characterize non-Gaussian diffusion processes. One approach to analyze HARDI data is to model the apparent diffusion coefficient (ADC) with higher order diffusion tensors (HODT). The diffusivity function is positive semi-definite. In the literature, some methods have been proposed to preserve positive semi-definiteness of second order and fourth order diffusion tensors. None of them can work for arbitrary high order diffusion tensors. In this paper, we propose a comprehensive model to approximate the ADC profile by a positive semi-definite diffusion tensor of either second or higher order. We call this model PSDT (positive semi-definite diffusion tensor). PSDT is a convex optimization problem with a convex quadratic objective function constrained by the nonnegativity requirement on the smallest Z-eigenvalue of the diffusivity function. The smallest Z-eigenvalue is a computable measure of the extent of positive definiteness of the diffusivity function. We also propose some other invariants for the ADC profile analysis. Performance of PSDT is depicted on synthetic data as well as MRI data.

PSDT can also be regarded as a conic linear programming (CLP) problem. Yinyu Ye and I investigated PSDT from the viewpoint of CLP. We characterize the dual cone of the positive semi-definite space tensor cone, and study the CLP formulation and duality of the positive semi-definite space tensor programming (STP) problem.

Numerical Methods for solving cone constrained variational inequality problems

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Abstract. We focus on the study of convergence analysis of numerical methods for variational inequality problems with cone constraints, including semismooth Newton methods and smoothing function methods for second-order cone constrained variational inequality problems and proximal point methods for variational inequality problems in Hilbert space. For second-order cone constrained variational inequality problems, the KKT system of a second order cone constrained variational inequality problem is transformed into a semismooth and later a smooth system of equations with the help of Fischer-Burmeister operators over second-order cones. The Clarke generalized differential of the semismooth mapping is presented. A modified Newton method with Armijo line search is proved to have global convergence with local superlinear rate of convergence under certain assumptions on the variational inequality problem. We also discuss the method based on the resolvent operator for general variational inequality in Hilbert space. A new monotonicity, M-monotonicity, is introduced. With the help of resolvent operator, equivalence between the variational inequality and the fixed point problem of a nonexpansive mapping is established. A proximal point algorithm is constructed to solve the fixed point problem, which is proved to have a global convergence. Furthermore, a convergent path Newton method, is given for the fixed point problems, enabling the proximal point algorithm implementable.

Equilibrium Problems and Nonlinear Operators in Optimization

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Abstract. Let E be a Banach space and let E^* be the dual space of E. Let $A \subset E \times E$ be a multi-valued operator with domain $D(A) = \{z \in E : Az \neq \emptyset\}$ and range $R(A) = \cup\{Az : z \in D(A)\}$. Then, $A \subset E \times E$ is called accretive if for each $x_i \in D(A)$ and $y_i \in Ax_i$, $i = 1, 2$, there exists $j \in J(x_1 - x_2)$ such that $\langle y_1 - y_2, j\rangle \geq 0$, where J is the duality mapping from E into 2^{E^*}. An accretive operator A is m-accretive if and only if $R(I + rA) = E$ for all $r > 0$. If $A \subset E \times E$ is m-accretive, then for each $r > 0$ and $x \in E$, we can define the resolvent $J_r\colon R(I + rA) \to D(A)$ by $J_r x = \{z \in E : x \in z + rAz\}$. A multi-valued operator $A\colon E \to E^*$ with domain $D(A) = \{z \in E : Az \neq \emptyset\}$ and range $R(A) = \cup\{Az : z \in D(A)\}$ is said to be monotone if $\langle x_1 - x_2, y_1 - y_2\rangle \geq 0$ for each $x_i \in D(A)$ and $y_i \in Ax_i$, $i = 1, 2$. A monotone operator A is said to be maximal if its graph $G(A) = \{(x, y) : y \in Ax\}$ is not properly contained in the graph of any other monotone operator. Let E be a reflexive, strictly convex and smooth Banach space and let $A\colon E \to 2^{E^*}$ be a monotone operator. Then, A is maximal if and only if $R(J + rA) = E^*$ for all $r > 0$. If $A \subset E \times E^*$ is a maximal monotone operator, then for $\lambda > 0$ and $x \in E$, we can consider the following resolvents:

$$J_\lambda x = \{z \in E : 0 \in J(z - x) + \lambda A(z)\}$$

and

$$Q_\lambda x = \{z \in E : Jx \in Jz + \lambda A(z)\}.$$

Further, if $B \subset E^* \times E$ is a maximal monotone operator, then for $\lambda > 0$ and $x \in E$, we can consider the resolvent

$$R_\lambda x = \{z \in E : x \in z + \lambda BJ(z)\}.$$

These four resolvents are important and have interesting properties. Let C be a nonempty closed convex subset of a Banach space E, let $f : C \times C \to \mathbb{R}$ be a bifunction. Then, we consider the following equilibrium problem:

$$\text{Find } z \in C \text{ such that } f(z, y) \geq 0, \ \forall y \in C. \tag{1}$$

The set of such $z \in C$ is denoted by $EP(f)$, i.e.,

$$EP(f) = \{z \in C : f(z, y) \geq 0, \ \forall y \in C\}.$$

The problem (1.1) is also important in the sence that it includes, as special cases, optimization problems, variational inequalities, minimax problems, Nash equilibrium problem in noncooperative games and others.

Our purpose in this talk is first to discuss nonlinear operators and nonlinear projections in Banach spaces which are related to the resolvents of m-accretive operators and maximal monotone operators. Some ones of these operators are new. Next, we discuss fixed point theorems and duality theorems for such nonlinear operators and nonlinear projections in Banach spaces. Further, using these properties, we prove strong convergence theorems for nonlinear operators with equilibrium problems in Banach spaces.

Collective Dynamics Decoding with Neural Circuit Labeling

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Abstract. The brain is considered to be a complex, self-organizing system; it consists of enormous numbers of interacting neurons and perpetually weaves its intricate web. Scientists believe that collective dynamics of the brain is deeply entwined with the neural circuits. But working out how neural circuits affect collective dynamics has been a great mystery. From the mathematical perspective, we wish to introduce a novel conception of "labeling neural circuits" and unlock how neural circuits affect collective dynamics. Of crucial importance, there are biological insights that follow from using this mathematical analysis.

An unified hybrid iterative method for zeros or fixed points of nonlinear operators

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Abstract. We propose an unified hybrid iterative algorithm obtained by coupling Φ-strongly pseudo-contractive operators and semigroups of demicontinuous non-Lipschitzian functions, and investigate its asymptotic behavior for approximating solutions of generalized variational inequality under mild control conditions on iteration parameters.

Splitting Methods for Monotone Operators and Their Applications in Signal Recovery

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Abstract. We will discuss iterative splitting algorithms for solving the inclusion problem

$$0 \in (A + B)x \qquad (*)$$

where A and B are maximal monotone operators in a Hilbert space H. These problems find applications in many applied areas, e.g., constrained least-squares problems, sparse regularization problems, signal recovery, and so on.

We shall present convergence results of some algorithms, like the nonlinear Peaceman-Rachford and Douglas-Rachford algorithms. Ergodic convergence will also be discussed. Possible applications will be mentioned.

An accelerated proximal gradient algorithm for nuclear norm regularized least squares problems

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Abstract. The affine rank minimization problem, which consists of finding a matrix of minimum rank subject to linear equality constraints, has been proposed in many areas of engineering and science. A specific rank minimization problem is the matrix completion problem, in which we wish to recover a (low-rank) data matrix from incomplete samples of its entries. A recent convex relaxation of the rank minimization problem minimizes the nuclear norm instead of the rank of the matrix. Another possible model for the rank minimization problem is the nuclear norm regularized linear least squares problem. This regularized problem is a special case of an unconstrained nonsmooth convex optimization problem, in which the objective function is the sum of a convex smooth function with Lipschitz continuous gradient and a convex function on a set of matrices. We propose an accelerated proximal gradient algorithm to solve this unconstrained nonsmooth convex optimization problem, and in particular, the nuclear norm regularized linear least squares problem. We report numerical results for solving large-scale randomly generated matrix completion problems. The numerical results suggest that our algorithm is efficient and robust in solving large-scale random matrix completion problems. In particular, we are able to solve random matrix completion problems with matrix dimensions up to 10^5 each in less than 10 minutes on a modest PC.