2010 Workshop on Nonlinear Analysis and Optimization

Department of Mathematics
National Taiwan Normal University

November 24-26, 20010

Sponsored by

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Mathematics Division, National Center for Theoretical Sciences (Taipei Office)

Organized by

Mau-Hsiang Shih and Jein-Shan Chen

Schedule of Programs

Place : M210, Mathematics Building

Table 1: November 24, Wednesday

	Chair	**Speaker**	Title
09:10 09:50	J-S Chen	D. Sun	An introduction to a class of matrix cone programming
09:50 10:30	J-S Chen	P-W Chen	A novel kernel correlation model with the correspondence estimation
10:50 11:30	J-S Jung	L-J Lin	Variational relation problems and equivalent forms of generalized Fan-Browder fixed point theorems with applications to Stampacchia equilibrium problems
11:30 12:10	J-S Jung	C-T Pang	Asymptotic stability of interval systems
		Lunch Break	
14:00 14:40	D-S Kim	J-S Jung	Some results on a general iterative method for k-strictly pseudo-contractive mappings
14:40 15:20	D-S Kim	S-N Lee	Large maximal IC-colorings for $K_{1,m,n}$
15:40 16:20	L-J Lin	J-L Ho	A combinatorial fixed point theorem in Boolean algebra
16:20 17:00	L-J Lin	D-S Kim	Duality relations for nondifferentiable fractional multiobjective programming problems

Table 2: November 25, Thursday

	Chair	**Speaker**	Title
09:10 09:50	J-S Chen	W. Takahashi	Fixed point and nonlinear ergodic theorems for generalized hybrid mappings
09:50 10:30	J-S Chen	X. Chen	Nonsmooth, nonconvex optimization with applications
10:50 11:30	J-S Chen	C-B Chua	Target-following framework for symmetric cone programming
11:30 12:10	J-S Chen	R-L Sheu	Duality and solutions for quadratic programming over single non-homogeneous quadratic constraint
		Lunch Break	
14:00 14:40	Takahashi	H-C Lai	Complex minimax programming with complex variables
14:40 15:20	Takahashi	S-C Huang	A hybrid extragradient method for asymptotically strict pseudo-contractions in the intermediate sense and inverse-strongly monotone mappings
15:40 16:20	H-C Lai	Y-A Hwang	Consistency of the Hirsch-index
16:20 17:00	H-C Lai	H-K Xu	Stochastic Féjer-monotonicity and its applications

Table 3: November 26, Friday

	Chair	**Speaker**	Title
09:10 09:50	J-S Chen	Y-L Chang	Stationary point conditions for the FB merit function associated with symmetric cones
10:00 10:40	J-S Chen	X. Miao	The column-sufficiency and row-sufficiency of the linear transformation on Hilbert spaces
10:50 11:30	J-S Chen	H-J Chen	Convergence rate analysis on interval-type algorithms for generalized fractional programming

Stationary point conditions for the FB merit function associated with symmetric cones

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Abstract. For the symmetric cone complementarity problem, we show that each stationary point of the unconstrained minimization reformulation based on the Fischer-Burmeister merit function is a solution to the problem, provided that the gradient operators of the mappings involved in the problem satisfy column monotonicity or have the Cartesian P_0-property. These results answer the open question proposed in the article appeared in Journal of Mathematical Analysis and Applications, vol. 355, pp. 195–215, 2009.

Convergence rate analysis on interval-type algorithms for generalized fractional programming

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Abstract. The generalized fractional programming is to minimize the largest of n ratios. Most algorithms for solving the generalized fractional programming are called the "Dinkelbach-type" which converts the original problem into a sequence of parametric subproblems. Interval-type algorithms differ from Dinkelbach-type in providing flexibility to select iterate parameters within the intervals, but the difficulty of estimating the convergence rate is to cope with the oscillating behavior of iterate parameters. In this talk, we will introduce the generic algorithm which can be regarded as a generalized version for interval-type algorithms and it creates a sequence of nested intervals containing the optimal value to the original problem whose lengths decrease to 0. The generic algorithm not only unifies various versions of the Dinkelbach-type algorithms, but give a stronger convergence result and the convergence as well as the convergence rate analysis are carried out through geometric observations and fundamental properties of convex analysis.

A novel kernel correlation model with the correspondence estimation

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Abstract. We present a novel multiple-linked iterative closest point method to estimate correspondences and the rigid/non-rigid transformations between point-sets or shapes. The estimation task is carried out by maximizing a symmetric similarity function, which is the product of the square roots of correspondences and a kernel correlation. The local mean square error analysis and robustness analysis are provided to show our method's superior performance to the kernel correlation method.

Nonsmooth, nonconvex optimization with applications

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Abstract. This talk will discuss nonsmooth, nonconvex optimization problems in stochastic equilibrium problems and l_2-l_p $(0 < p < 1)$ minimization problems, as well as their applications in transportation planning, signal reconstruction and variable selection. In particular, this talk will present our recent results in the following two parts.

(i) We reformulate the stochastic variational inequality problems as expected residual minimization problems using residual functions of variational inequalities. Mathematical theorems and practical examples of traffic assignments show that the reformulations are robust and reliable in uncertain environments.

(ii) We derive a lower bound theory for nonzero entries in every local minimizer of the l_2-l_p minimization problems. This theory shows clearly the relationship between the sparsity of the solution and the choice of parameters in the model. We prove global convergence of the l_1 reweighted minimization algorithm and uniqueness of solution under the truncated null space property which is weaker than the restricted isometry property.

Target-following framework for symmetric cone programming

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Abstract. The first target-following algorithm was given by Shinji Mizuno in 1992 for linear complementarity problems, using the notion of delta sequences. The delta sequence is a sequence of targets in the 'v-space' that lead the strictly feasible primal-dual solutions towards optimality. In this talk, I will present a generalization of the target-following framework to symmetric cone programming.

A Combinatorial fixed point theorem in Boolean algebra

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Abstract. We propose to answer the Jacobian conjecture in boolean algebra. The boolean analogue of the Jacobian problem in 0,1? has been proved: if a map from $\{0,1\}$? to itself defines a boolean network has the property that all the boolean eigenvalues of the discrete Jacobian matrix of this map evaluated at each element of $\{0,1\}$?are zero, then it has a unique fixed point. We propose extending this result to any map F from the product space X of n finite boolean algebras to itself.

Keywords: Jacobian conjecture; Combinatorial fixed point theorem, Discrete Boolean eigenvalues; Finite boolean algebras.

A hybrid extragradient method for asymptotically strict pseudo-contractions in the intermediate sense and inverse-strongly monotone mappings

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Abstract. This talk is devoted to investigating a new hybrid extragradient method for an asymptotically strict pseudo-contraction in the intermediate sense S and an inverse-strongly monotone mapping A in a Hilbert space. The main purpose is to use this iteration method to generate a sequence to approximate a common element of the fixed point set of S and the solution set of the variational inequality problem for A. Weak convergence and strong convergence for the related sequences are established with respective iteration processes.

Consistency of the Hirsch-index

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Abstract. The Hirsch-index is an index for measuring and comparing the output of researchers. Under the condition of monotonicity, Woeginger (2008) provides a characterization of the Hirsch-index by three axioms. Replacing monotonicity by expansion consistency, we characterize the Hirsch-index by only two of Woeginger's axioms. Besides, we also introduce an axiom contraction consistency. It is a dual viewpoint of expansion consistency. Based on contraction consistency, an additional characterization of the Hirsch-index is reported.

Some results on a general iterative method for k-strictly pseudo-contractive mappings

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Abstract. In this paper, we introduce a new general iterative scheme for a k-strictly pseudo-contractive mapping related to an operator F which is κ-Lipschizian and η-strongly monotone and then prove that under certain different control conditions, the sequence generated by the proposed iterative scheme converges strongly to a fixed point of the mapping, which solves a variational inequality related to the operator F. Additional results of main results are also obtained. Our results substantially improve and develop the corresponding ones announced by many authors recently.

Key words. k-strictly pseudo-contractive mapping; Nonexpansive mapping; Fixed points; Contraction; κ-Lipschizian and η-strongly monotone operator; Hilbert space; Variational inequality.

2000 *Mathematics Subject Classification.* 47H09, 47H10, 47J20, 47J25, 49M05.

Duality relations for nondifferentiable fractional multiobjective programming problems

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Abstract. In this talk, we consider the nondifferentiable multiobjective fractional programming problem involving support functions and cone constraints. For this problem, Wolfe and Mond-Weir type duals are proposed. We establish weak and strong duality theorems for a weakly efficient solution by using generalized convexity conditions. In addition, we introduce a pair of nondifferentiable multiobjective symmetric dual problems with cone constraints over arbitrary closed convex cones. Weak, strong and converse duality theorems are established under suitable generalized convexity conditions for a weakly efficient solution. As special cases of our duality relations are given.

2000 *Mathematics Subject Classification.* 90C29; 90C32; 90C46.

Key words and phrases. Multiobjective fractional programming, support functions, optimality conditions, duality theorems.

Complex minimax programming with complex variables

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Abstract. Consider a nondifferentiable minimax fractional programming problems with complex variables as the following form:

$$(\mathrm{FP}_c) \qquad \min_{\zeta\in X}\max_{\eta\in Y} \frac{Re\ [f(\zeta,\eta)+(z^HAz)^{1/2}]}{Re\ [g(\zeta,\eta)-(z^HBz)^{1/2}]} \quad \text{s.t.} \quad X=\{\zeta=(z,\overline{z})\in\mathbb{C}^{2n} \mid -h(\zeta)\in S\}$$

where Y is a specified compact subset of $\mathbb{C}^{2m}$, A and B are positive semidefinite Hermitian matrices in $\mathbb{C}^{n\times n}$, S is a polyhedral cone in $\mathbb{C}^p$, $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are continuous on $\mathbb{C}^n\times\mathbb{C}^m$, and for each $\eta\in Y$, $f(\cdot,\eta)$, $g(\cdot,\eta)$ and $h(\cdot)$ are analytic functions.

In this talk, the duality models of (FP_c) are established and the duality theorems related to problem (FP_c) are proved with nonduality gap under some conditions.

References

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Large maximal IC-colorings for $K_{1,m,n}$

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Abstract. The *IC*-index $M(K_{1,m,n})$ of the class $K_{1,m,n}$ of all complete tripartite graphs and its corresponding maximal colorings are obtained. We prove that $M(K_{1,1,n}) = 3 \cdot 2^n + 1$ $(n \geq 1)$ and $M(K_{1,m,n}) = 13 \cdot 2^{m+n-3} - 2^{m-2} + 2$ $(n \geq m \geq 3)$ and that, up to *IC*-equivalence, the classes $K_{1,1,1}$, $K_{1,1,2}$, $K_{1,1,n}$ $(n \geq 3)$, $K_{1,2,2}$, $K_{1,2,n}$ $(n \geq 3)$ and $K_{1,m,n}$ $(n \geq m \geq 3)$ have exactly one, four, two, six, four and one maximal colorings, respectively.

Variational relation problems and equivalent forms of generalized Fan-Browder fixed point theorems with applications to Stampacchia equilibrium problems

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Abstract. In this paper, we study the existence theorems of solution for variational relation problems. From the existence theorems of solution for variational relation problems, we study equivalent forms of generalized Fan-Browder fixed point theorem, existence theorems of solutions for Stampacchia vector equilibrium problems and generalized Stampacchia vector equilibrium problems. Our results contains many orginal results and have many applications in Nonlinear Analysis.

The column-sufficiency and row-sufficiency of the linear transformation on Hilbert spaces

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Abstract. In this talk, firstly, we introduce the concepts of the column-sufficiency and row-sufficiency of the linear transformation on Hilbert space. Secondly, we show that the row-sufficiency of T is equivalent to the existence of the solution of the linear complementarity problem under an operator commutative condition; moreover, the column-sufficiency along with cross commutative property is equivalent to the convexity of the solution set of the linear complementarity problem. In our analysis, the properties of the Jordan product and the Lorentz cone in Hilbert space play important roles.

Asymptotic stability of interval systems

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Abstract. Previous works about the convergence of the powers of interval matrices have focused on the iteration of a single interval matrix. But in robust stability analysis of uncertain systems, there is associated with a set of coupled interval matrices. The most basic issue is the the asymptotic stability of a set of interval matrices. Here we introduce the notion of simultaneous Schur stability by linking the concepts of the majorant and the joint spectral radius, and prove the asymptotic stability of a set of interval matrices governed by simultaneous Schur stability. The present result may lead to the stability analysis of discrete dynamical interval systems.

Duality and solutions for quadratic programming over single non-homogeneous quadratic constraint

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Abstract. The quadratic programming over one quadratic constraint (QP1QC) was mostly studied under certain constraint qualifications such as the Slater condition. In this talk, we relax the assumption to cover more general cases when the two matrices from the objective and the constraint functions can be simultaneously diagonalizable via congruence. Under such an assumption, the nonconvex (QP1QC) problem can be classified into three types: (i) a unbounded below problem; or (ii) a unconstrained quadratic problem; or (iii) one with a feasible dual problem with no duality gap. In other words, the (QP1QC) problem is a "good" non-convex programming. We can explain by showing that the (QP1QC) problem is indeed equivalent to a linearly constrained convex problem, which happens to be dual of the dual of itself.

An introduction to a class of matrix cone programming

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Abstract. In this talk, we shall introduce a class of linear conic programming (which we call matrix cone programming or MCP) involving the epigraphs of five commonly used matrix norms and the well studied symmetric cone. MCP has recently found many important applications, for example, in nuclear norm relaxations of affine rank minimization problems. In order to make the defined MCP tractable and meaningful, we must first understand the structure of these epigraphs. So far, only the epigraph of the Frobenius matrix norm, which can be regarded as a second order cone, has been well studied. Here, we take an initial step to study several important properties, including its closed form solution, calm Bouligand-differentiability and strong semismoothness, of the metric projection operator over the epigraph of the l_1, l_∞, spectral or operator, and nuclear matrix norm, respectively. These properties make it possible to apply augmented Lagrangian methods, which have recently received a great deal of interests due to their high efficiency in solving large scale semidefinite programming, to this class of MCP problems. The work done on MCP is far from comprehensive. Rather it is intended as a starting point to call for more insightful research on MCP so that it can serve as a basic tool to solve more challenging convex matrix optimization problems in years to come. [This is a joint work with Chao DING and Kim Chuan TOH]

Fixed point and nonlinear ergodic theorems for generalized hybrid mappings

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Abstract. Let H be a real Hilbert space and let C be a nonempty subset of H. Then a mapping $T : C \to H$ is said to be nonexpansive if

$$\|Tx - Ty\| \leq \|x - y\|$$

for all $x, y \in C$. The set of fixed points of T is denoted by $F(T)$. In 1975, Baillon proved the following first nonlinear ergodic theorem in a Hilbert space.

Theorem 1. *Let C be a nonempty closed convex subset of H and let $T : C \to C$ be a nonexpansive mapping with $F(T) \neq \emptyset$. Then, for any $x \in C$, $S_n x = \frac{1}{n}\sum_{k=0}^{n-1} T^k x$ converges weakly to an element $z \in F(T)$.*

An important example of nonexpansive mappings in a Hilbert space is a firmly nonexpansive mapping. Recently, Kohsaka and Takahashi, and Takahashi introduced the following mappings which are deduced from a firmly nonexpansive mapping in a Hilbert space. A mapping $T : C \to H$ is called nonspreading if

$$2\|Tx - Ty\|^2 \leq \|Tx - y\|^2 + \|Ty - x\|^2$$

for all $x, y \in C$. A mapping $T : C \to H$ is called hybrid if

$$3\|Tx - Ty\|^2 \leq \|x - y\|^2 + \|Tx - y\|^2 + \|Ty - x\|^2$$

for all $x, y \in C$.

In this talk, we first introduce a broad class of mappings $T : C \to H$ called generalized hybrid such that for some $\alpha, \beta \in \mathbb{R}$,

$$\alpha\|Tx - Ty\|^2 + (1 - \alpha)\|x - Ty\|^2 \leq \beta\|Tx - y\|^2 + (1 - \beta)\|x - y\|^2$$

for all $x, y \in C$. Such a class contains the classes of nonexpansive mappings, nonspreading mappings, and hybrid mappings in a Hilbert space. Next, we prove fixed point and nonlinear ergodic theorems for generalized hybrid mappings in a Hilbert space. Finally, we deal with two strong convergence theorems for these nonlinear mappings in a Hilbert space.

Stochastic Féjer-monotonicity and its applications

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Abstract. Let H be a real Hilbert space and let C be a nonempty closed convex subset of H. Let $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{n=1}^{\infty}, \mathbb{P})$ be a filtered probability space. Let $\{x_n\}_{n=1}^{\infty}$ be a sequence of random variables taking values in H which is adapted (i.e., x_n is $\mathcal{F}_n$-measurable for each n). We say that $\{x_n\}_{n=1}^{\infty}$ is stochastically quasi-Féjer-monotone with respect to C if there exists a sequence $\{\varepsilon_n\}_{n=1}^{\infty}$ of nonnegative random variables, with $\sum_{n=1}^{\infty} \varepsilon_n < \infty$ and satisfying the property

$$\mathbb{E}\left[\|x_{n+1} - x\|^2 | \mathcal{F}_n\right] \leq \|x_n - x\|^2 + \varepsilon_n \quad \text{(a.s.)}, \ n \geq 1, \ x \in C.$$

In this talk, I will present some properties of stochastically quasi-Féjer-monotone sequences. I will also discuss applications to optimization, in particular, the stochastic subgradient algorithm for solving the minimization problem $\min_{x \in C} f(x)$, where $f : H \to \mathbb{R}$ is a convex function.