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Fixed Points of Asymptotically Strict Pseudo-Contractions in The Intermediate Sense and Variational Inequalities

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Abstract. In this talk we consider a new hybrid extragradient method to find a common element of the set of fixed points of an asymptotically strictly pseudo-contractive mapping in the intermediate sense and the set of solutions of the variational inequality problem for an inverse-strongly monotone mapping in a Hilbert space. Using this iteration, we obtain a necessary and sufficient condition for a strong convergence theorem.

Smoothing-Type Algorithms for the SCCP

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Abstract. This talk is about the smoothing-type algorithm for solving the symmetric cone complementarity problem (SCCP), which mainly includes the following issues: (1) how to design a smoothing-type algorithm to solve the monotone SCCP, including reformulation of the problem, solvability of Newton equations, and convergence of the algorithm; (2) what's the difference between the smoothing-type algorithm for solving the nonmonotone SCCP and the traditional complementarity problem with a nonmonotone mapping; (3) what's the regularized technique and why it is used in smoothing-type algorithms, etc.. In addition, some issues to be further investigated are also introduced.

Marginal monotonicity solution of NTU games

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Abstract. In the presence of independence of undominating alternatives and continuity, we show that the symmetric egalitarian solution is the unique symmetric solution satisfying marginal monotonicity.

Keywords: symmetric egalitarian solution; marginal monotonicity.

JEL Classification Numbers: C71.

Realcompactness and related Banach-Stone theorem

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Abstract. In this talk, we consider the linear isometries T between $C_b(X, E)$ and $C_b(Y, F)$, where X and Y are realcompact topological spaces and E and F are Banach spaces. We will give some sufficient conditions such that T is a weighted composition operator.

Second-order cone constrained variational inequality problems

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Abstract. For second-order cone constrained variational inequality problems, the KKT system of a second order cone constrained variational inequality (SOCCVI) problem is transformed into a semismooth and later a smooth system of equations with the help of Fischer-Burmeister operators over second-order cones. The Clarke generalized differential of the semismooth mapping is presented. A modified Newton method with Armijo line search is proved to have global convergence with local superlinear rate of convergence under certain assumptions on the variational inequality problem. Furthermore, we consider using the neural networks to efficiently solve the second-order cone constrained variational inequality problems. In addition, we consider using the neural networks to efficiently solve the second-order cone constrained variational inequality problem. More specifically, two kinds of neural networks are proposed to deal with the Karush-Kuhn-Tucker (KKT) conditions of the SOCCVI problem. The first neural network uses the Fischer-Burmeister (FB) function to achieve an unconstrained minimization which is a merit function of the Karush-Kuhn-Tucker equation. We show that the merit function is a Lyapunov function and this neural network is asymptotically stable. The second neural network is introduced for solving a projection formulation whose solutions coincide with the KKT triples of SOCCVI problem. Its Lyapunov stability and global convergence are proved under some conditions. Simulations show the effectiveness of the proposed neural networks.

Keywords: Second-order cone; Variational inequality; Fischer-Burmeister function; Newton method; Neural networks; Lyapunov stable

Nonlinear Analytic Methods for Linear Contractive Mappings in Banach Spaces

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Abstract. Let E be a real Banach space and let C be a closed convex subset of E. For a mapping $T : C \to C$, we denoted by $F(T)$ the set of fixed points of T. A mapping $T : C \to C$ is called nonexpansive if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in C$. In particular, a nonexpansive mapping $T : E \to E$ is called contractive if it is linear, that is, a linear contactive mapping $T : E \to E$ is a linear operator satisfying $\|T\| \leq 1$. We know a well-known weak convergence theorem by Mann's iteration for nonexpansive mappings in a Hilbert space H: Let $T : C \to C$ be a nonexpansive mapping with $F(T) \neq \emptyset$ and define a sequence $\{x_n\}$ in C by $x_1 = x \in C$ and

$$x_{n+1} = \alpha_n x_n + (1 - \alpha_n)Tx_n$$

for all $n \in \mathbb{N}$, where $\{\alpha_n\}$ is a real sequence in $[0, 1]$ such that $\sum_{n=1}^{\infty} \alpha_n(1 - \alpha_n) = \infty$. Then, $\{x_n\}$ converges weakly to an element z of $F(T)$, where $z = \lim_{n\to\infty} Px_n$ and P is the metric projection of H onto $F(T)$.

In 1979, Reich extended such a theorem to a uniformly convex Banach space with a Fréchet differentiable norm. However, we have not known whether z is characterized under any projections in a Banach space. Recently, using nonlinear analytic methods, Takahashi and Yao solved such a problem for positively homogeneous nonexpansive mappings in a Banach space. In 1938, Yosida also proved the following mean ergodic theorem for linear bounded operators: Let E be a real Banach space and let T be a linear operator of E into itself such that there exists a constant C with $\|T^n\| \leq C$ for $n \in \mathbb{N}$, and T maps the closed unit ball of E into a weakly compact subset of E. Then, for each $x \in E$, the Cesàro means

$$S_n x = \frac{1}{n} \sum_{k=1}^{n} T^k x$$

converge strongly as $n \to \infty$ to a fixed point of T.

In this talk, motivated by these theorems, we study nonlinear analytic methods for linear contractive mappings in Banach spaces and obtain some new weak and strong convergence theorems for linear or nonlinear operators in Banach spaces.