2011 Workshop on Nonlinear Analysis and Optimization

Department of Mathematics
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Organized by

Mau-Hsiang Shih and Jein-Shan Chen

Schedule of Programs
Place : M210, Mathematics Building

Table 1: November 16, Wednesday

	Chair	**Speaker**	Title
09:00 09:40	M-H Shih	W. Takahashi	Nonlinear Ergodic Theorems without convexity for nonlinear mappings in Banach spaces
09:40 10:20	M-H Shih	S. Akashi	Classification problem of one dimensional chaotic dynamical systems
		Tea Break	
10:40 11:20	H-C Lai	L-J Lin	Fixed Point Theorems of (a, b)-monotone mappings in Hilbert spaces
11:20 12:00	H-C Lai	J-S Jung	A general iterative scheme for inverse-strongly monotone mappings and strictly pseudo-contractive mappings
		Lunch Break	
14:00 14:40	D-S Kim	H-C Lai	Saddle value functions and Minimax Theorems on two-person zero-sum dynamic fractional game
14:40 15:20	D-S Kim	Z. Peng	An alternating direction method for Nash equilibrium of two-person games with alternating offers
		Tea Break	
15:40 16:20	L-J Lin	S. Plubtieng	Some existence results of solutions for generalizations of Ekeland-type variational principle and a system of general variational inequalities
16:20 17:00	L-J Lin	D. Dhompongsa	Nonexpansive retracts and common fixed points

Table 2: November 17, Thursday

	Chair	**Speaker**	Title
09:00 09:40	J-S Chen	L. Tuncel	Local quadratic convergence of polynomial-time interior-point methods for convex optimization problems
09:40 10:20	J-S Chen	C-J Lin	Optimization methods for L_1-regularized problems
		Tea Break	
10:40 11:20	J-S Chen	R-L Sheu	On the double well potential problem
11:20 12:00	J-S Chen	P-W Chen	Mass transport problem and its applications
		Lunch Break	
14:00 14:40	R-L Sheu	X. Yuan	Customized proximal point algorithms for separable convex programming
14:40 15:20	R-L Sheu	C-H Yeh	Axiomatic and strategic justifications for the constrained equal benefits rule in the airport problem
		Tea Break	
15:40 16:20	S. Akashi	J-Y Lin	Joint replenishment problem under conditions of permissible delay in payments
16:20 17:00	S. Akashi	D-S Kim	A new approach to characterize solution set of a nonconvex optimization problem

Table 3: November 18, Friday

	Chair	**Speaker**	Title
09:10 09:50	J-S Chen	H-J Chen	Augmented Lagrange primal-dual approach for generalized fractional programs
10:00 10:40	J-S Chen	C-H Huang	Maximal elements of majorized Q_α-condensing mappings
10:50 11:30	J-S Chen	S-Y Hsu	Asymptotic index dictates the stability for discrete nonautonomous linear systems

Classification problem of one dimensional chaotic dynamical systems

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Abstract. Almost all of the technical methods, which are used in the theory of nonlinear dynamical systems, originated in the theory of one dimensional dynamical systems. In this talk, a relation between nonlinear ergodic theorems, which are developed by Prof. Takahashi and other researchers, and the asymptotic behavior of one dimensional dynamical systems will be delivered, and moreover, application of this relation to Collatz-Kakutani conjecture will be shown.

Augmented Lagrange Primal-Dual Approach for Generalized Fractional Programs

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Abstract. In this talk, we will introduce a primal-dual approach for solving the generalized fractional program :

$$(\mathrm{P})\ \lambda^* = \min_{x\in X}\max_{i\in\Lambda}\{\frac{f_i(x)}{g_i(x)}\}$$

where X is convex and compact in $\mathbb{R}^n$, $\Lambda = \{1, 2, \cdots, r\}$, $\{f_i(x)\}_{i\in\Lambda}$, and $\{g_i(x)\}_{i\in\Lambda}$ are two finite collections of continuous functions on X such that $\min_{x\in X}\min_{i\in\Lambda} g_i(x) > 0$. The traditional approach is to solve (P) through a sequence of parametric subproblems :

$$(\mathrm{P}_{\lambda_k})\ F(\lambda_k) = \min_{x\in X}\max_{i\in\Lambda}\{f_i(x) - \lambda_k g_i(x)\}.$$

The outer iteration of the Augmented Lagrange Primal-Dual Algorithm is a kind of interval-type Dinkelbach algorithm, while the augmented Lagrange method is adopted for solving the inner min-max subproblems.

The Lagrange dual of (P_{λ_k}) can be formulated as

$$(D_{\lambda_k})\quad G(\lambda_k) = \max_{y\in\Delta}\min_{x\in X}\sum_{i=1}^{r}[y_i(f_i(x) - \lambda_k g_i(x))]$$

where $\Delta = \{y \in \mathbb{R}^r | \sum_{i=1}^r y_i = 1,\ y_i \geq 0,\ i \in \Lambda\}$ is the $r-$dimensional simplex and we can obtain a no-duality-gap under additional convex assumptions. The augmented Lagrange method attaches a set of artificial variables as well as their corresponding Lagrange multipliers to the min-max subproblem (P_{λ_k}) :

$$\min_{x\in X}\inf_{u\leq 0}\max_{i\in\Lambda}\{f_i(x) - \lambda_k g_i(x) - u_i\}$$

with an auxiliary variable u, additional constraints $u \leq 0$; and a new set of Lagrangian multipliers $\mu \in \Delta$. By iterating μ in the following formula

$$\mu_i^{l+1} = \frac{\mu_i^l \exp(f_i(x^{l+1}) - \lambda_k g_i(x^{l+1}))}{\sum_{i=1}^r [\mu_i^l \exp(f_i(x^{l+1}) - \lambda_k g_i(x^{l+1}))]}$$

and solve the convex minimization problem

$$\min_{x \in X} \log\{\sum_{i=1}^r [\mu_i^{l+1} \exp(f_i(x) - \lambda_k g_i(x))]\},$$

we will show that the primal optimal solution of (P_{λ_k}) and the dual optimal solution of (D_{λ_k}) can be solved simultaneously. As a result, both the primal and the dual information is available for updating the iterate points and the min-max subproblem is then reduced to a sequence of convex minimization problems.

Mass transport problem and its applications

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Abstract. The quadratic Wasserstein distance is an important distance and it amazingly arises in various applications. In this talk, we will study its application in imaging registration. In this paper, we present a new registration method for solving point set matching problems based on mass transport, i.e., minimizing quadratic Wasserstein distance. Roughly speaking, the method utilizes a global affine transform and a local curl-free transform. The affine transform is estimated by the first two moments of point sets, which is equivalent to the asymptotic transform in the kernel correlation method as the kernel scale approaches infinity. The curl-free transform is achieved by optimizing some kernel correlation function weighted by a square root of a pair of correspondence matrices. We apply this method to match two sets of pulmonary vascular tree branch points whose displacement is caused by the lung volume changes of the same human subject. Nearly perfect match performances on six human subjects verdict the effectiveness of this model. For theoretical interests, we also study the consistency property between the discrete model and the continuous model. This is a joint work with Ching-Long Lin and I-Liang Chern.

Nonexpansive retracts and common fixed points

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Abstract. In this talk, we first consider a commuting family of nonexpansive mappings, one of which is multi-valued, by showing that their set of common fixed points is a nonexpansive retract of a given weakly compact convex domain. Then we will consider, by using a Bruck's result (in 1973), an iteration to compute a common fixed point of a countably many number of single-valued non-expansive mappings and a multivalued nonexpansive mapping on a strictly convex Banach space. The corresponding result is also considered for the CAT(0) space setting.

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Asymptotic index dictates the stability for discrete nonautonomous linear systems

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Abstract. When analyzing models of some economical phenomena, one may possibly encounter nonautonomous systems in which the equilibrium may change with time. In this talk, we will describe the stability for nonautonomous systems with moving equilibriums. Accordingly, the definition of stability for autonomous cases is extended to nonautonomous cases. We will formulate a quantity called asymptotic index of a sequence of matrices and show that the stability for nonautonomous systems can be characterized by the asymptotic index.

Maximal Elements of majorized Q_α-Condensing Mappings

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Abstract. An H-space is a topological space X, together with a family $\{\Gamma_D\}$ of some nonempty contractible subsets of X indexed by $D \in \langle X \rangle$ such that $\Gamma_D \subset \Gamma_{D'}$ whenever $D \subset D'$. An H-space X is called an $l.c.$-space, if X is an uniform space whose topology is induced by its uniformity $\mathcal{U}$, and there is a base $\mathcal{B}$ consisting of symmetric entourages in $\mathcal{U}$ such that for each $V \in \mathcal{B}$, the set $V(E) := \{y \in X \mid (x, y) \in V \text{ for some } x \in E\}$ is H-convex whenever E is H-convex. In this talk, we consider a family of $\mathcal{L}_{\pi_\alpha}$-majorized Q_α-condensing mappings $T_\alpha : X \longrightarrow 2^{X_\alpha}$, where each X_α is an $l.c.$-space with precompact polytopes, and $X := \prod_{\alpha \in I} X_\alpha$. First, we establish a new existence theorem of maximal elements for the product mapping $T := \prod_{\alpha \in I} T_\alpha$. Further, we prove that the family $\{T_\alpha \mid \alpha \in I\}$ admits a common maximal element under the mild condition that each $\{x \mid T_\alpha(x) \neq \emptyset\}$ is compactly open.

Keywords. $l.c.$-space, Q_α-condensing mapping, maximal element, $\mathcal{L}_\theta$-majorized.

2000 AMS subject classifications. 47H04, 52A99, 54H25.

A general iterative scheme for inverse-strongly monotone mappings and strictly pseudo-contractive mappings

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Abstract. In this talk, we introduce a general iterative scheme for finding a common element of the set of solutions of variational inequality problem for an inverse-strongly monotone mapping and the set of fixed points of a strictly pseudo-contractive mapping in a Hilbert space and then establish strong convergence of the sequence generated by the proposed iterative scheme to a common element of the above two sets under suitable control conditions, which is a solution of a certain variational inequality. Our results develops and complements the corresponding results given by many authors recently in this area.

A new approach to characterize solution set of a nonconvex optimization problem

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Abstract. In this talk, we consider the following problem

$$\begin{array}{lll} \text{(P)} & \text{Minimize} & f(x) \\ & \text{subject to} & f_i(x) \leq 0,\ i \in M := \{1, 2, \cdots, m\}, \\ & & x \in C, \end{array}$$

where C is a closed convex subset of X, $f, f_i : X \to \mathbb{R}, i \in M$, are locally Lipschitz functions on a Banach space X. As a different approach, we prove that Lagrange function corresponding to a fixed Lagrange multiplier is constant on a set containing its solution set of (P). Moreover, we establish that Lagrange function corresponding to some given multiplier is constant on a subset including the solution set. The relations between the subsets concerning the dual feasible set and the set of saddle points of the Lagrange function are investigated.

2000 *Mathematics Subject Classification.* 90C26, 90C30, 90C46.

Key words and phrases. Nonconvex programming; Lagrange function; solution sets; saddle points.

Saddle Value Functions and Minimax Theorems on Two-Person Zero-Sum Dynamic Fractional Game

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Abstract. Problem motivated from Ky Fan's minimax theorem:

Let X and Y be any sets (not necessary topology, not be linear). For a real valued function $f : X \times Y \longrightarrow \mathbb{R}$, Ky Fan proved that

$$\min_{x\in X}\max_{y\in Y} f(x,y) = \max_{y\in Y}\min_{x\in X} f(x,y)$$

holds under certain conditions.

We consider a two-person zero-sum dynamic game of fractional type:

$$\phi(x,y) = \frac{f(x,y)}{g(x,y)}, \quad (x,y) \in X \times Y.$$

Purpose of this talk is to show the minmax theorem holds for the universal strategy spaces X and Y in the sense of measurable transition probabilities. Precisely, we show that

$$\inf_{x\in X}\sup_{y\in Y} W(x,y) = \sup_{y\in Y}\inf_{x\in X} W(x,y)$$

where $W(x,y) = \dfrac{U(x,y)}{V(x,y)}$, $(x,y) \in X \times Y$, and U, V are the conditional expectations regarded as the reward functionals for players I and II after they have chosen their strategies $x \in X$ and $y \in Y$ in the infinitely many story H_∞ in the game system.

Optimization methods for L_1-regularized problems

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Abstract. Recently L_1-regularized problems are widely used in many applications. For example, L_1-regularized least square regression is useful for compressed sensing and other signal processing applications. L_1-regularized logistic regression and support vector machines can be applied to classification and feature selection. All these L_1-regularized applications involve a non-differentiable optimization problem. In the first part of this talk, we briefly review the state of the art software for L_1-regularized classification. Next, we present theoretical and implementation details of one or two methods. Finally, we discuss some interesting differences between L_1-regularized classification and regression.

Joint Replenishment Problem Under conditions of Permissible Delay in Payments

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Abstract. In the real world, the suppliers often provide a permissible period such that the retailer only have to settle the payment before this period. Before the settlement of payments, the retailers can sell the goods, accumulate revenue and earn the interests. But, if the payment is settled after this period, the retailers have to be charged some interests. The traditional JRP models concern how to determine lot sizes and to schedule replenishment times for products, but their objective functions doesn't include the earned or charged interest. In this paper, we state the mathematical model the JRP under the conditions of permissible delay in payments. We investigate that the objective function of this model is a piecewise-convex function and use this truth to find all local optimal solutions. Hence we can obtain the global optimal solutions. Besides the theoretical analysis, we also organize an efficient algorithm for finding the global optimal solutions and implement it on a numerical example.

Fixed Point Theorems of (a, b)-Monotone Mappings in Hilbert Spaces

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Abstract. We propose a new class of nonlinear mappings, called (a, b)-monotone mappings, and show that this class of nonlinear mappings contains nonspreading mappings, hybrid mappings, firmly nonexpansive mappings and $(a_1, a_2, a_3, k_1, k_2)$-generalized hybrid mappings with $a_1 < 1$. We also give an example to show that a (a, b)-monotone mapping is not necessary to be a quasi-nonexpansive mapping. We establish an existence theorem of fixed points and the demiclosed principle for the class of (a, b)-monotone mappings. As a special case of our result, we give an existence theorem of fixed points for $(a_1, a_2, a_3, k_1, k_2)$-generalized hybrid mappings with $a_1 < 1$. We also consider Mann's type weak convergence theorem and CQ type strong convergence theorem for (a, b)-monotone mappings. An example is given to show the Mann's type weak convergence theorem for the (a, b)-monotone mappings

An alternating direction method for Nash equilibrium of two-person games with alternating offers

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Abstract. In this talk, we propose a method for finding a Nash equilibrium of two-person games with alternating offers. The proposed method is referred to as inexact proximal alternating direction (inPAD) method. In the inPAD method, the idea of alternating direction method simulates alternating offers in the game, while the inexactness matches to asymmetry information and limited individual rationality in practice. The convergence of the proposed inPAD method is proved under some suitable conditions. Numerical tests show that the inPAD method is superior to some existing projection-like methods in literature.

Keywords: Computational game theory; Nash equilibrium; inexact proximal point method; alternating direction method.

Some existence results of solutions for generalizations of Ekeland-type variational principle and a system of general variational inequalities

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Abstract. In this talk, we first introduce the concept of a Q-function defined on a quasi-metric space, which generalizes the notion of a w-distance, and prove Ekeland-type variational principles in the setting of quasi-metric spaces with a Q-function. We also present an equilibrium version of the Ekeland-type variational principle in the setting of quasi-metric spaces with a Q-function. Secondary, we present some existence results for a new system of general variational inequalities of Stampacchia type and of Minty type, respectively. As consequences, some well known classic results from the literature are obtained. Finally, we conclude our paper by emphasizing the results that were obtained.

On the Double Well Potential Problem

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Abstract. The double well potential problem is to optimise a special type of multi-variate polynomial of degree 4. It appears in many applications such as in solid mechanics, in Landau-Ginzburg theory of the second order ferroelectric transformations, and in the model describing hydrogen dynamics in carboxylic acids, etc. In this talk, we characterize all the local/global minimizers and maximizers of the double-well potential problem. It is proven that for the nonsingular case there exists at most one local-nonglobal minimizer and at most one local maximizer. The local maximizer is "surrounded" by local minimizers in the sense that its weighted norm is strictly smaller than that of any local minimizer. We establish necessary and sufficient conditions for the global minimizer, reveal the hidden convex nature of the problem, and develop an efficient algorithm for solving it.

Nonlinear Ergodic Theorems without convexity for nonlinear mappings in Banach spaces

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Abstract. In this talk, we introduce the concept of attractive points of nonlinear mappings in a Banach space and obtain some fundamental properties for the points. Using these results, we prove attractive point theorems for nonlinear mappings in a Banach space. Using these results, we prove nonlinear ergodic theorems without convexity for nonlinear mappings in a Banach space. These results extend attractive point theorems which were recently proved by Takahashi and Takeuchi in Hilbert spaces to Banach spaces.

Local quadratic convergence of polynomial-time interior-point methods for convex optimization problems

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Abstract. We propose new path-following predictor-corrector schemes for solving convex optimization problems in conic form. The main structural property used in our analysis is the logarithmic homogeneity of self-concordant barrier functions. Even though our analysis has primal and dual components, our algorithms work with the dual iterates only, in the dual space. Our algorithms converge globally as the current best polynomial-time interior-point methods. In addition, our algorithms have the local quadratic convergence property under some mild assumptions. The algorithms are based on an easily computable gradient proximity measure, which ensures an automatic transformation of the global linear rate of convergence to the local quadratic one under some mild assumptions. Our step-size procedure for the predictor step is related to the maximum step size (the one that takes us to the boundary). It appears that in order to obtain local superlinear convergence, we need to tighten the neighborhood of the central path proportionally to the current duality gap.

This talk is based on joint work with Yu. Nesterov.

Axiomatic and strategic justifications for the constrained equal benefits rule in the airport problem

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Abstract. We consider the "airport problem", which is concerned with sharing the cost of an airstrip among agents who need airstrips of different lengths. We investigate the implications of two properties, Left-endpoint Subtraction (LS) bilateral consistency and LS converse consistency, in the airport problem. First, on the basis of the two properties, we characterize the constrained equal benefits rule, which equalizes agents' benefits subject to no one receiving a subsidy. Second, we introduce a 2-stage extensive form game that exploits LS bilateral consistency and LS converse consistency. We show that there is a unique subgame perfect equilibrium outcome of the game and moreover, it is the allocation chosen by the constrained equal benefits rule.

Customized proximal point algorithms for separable convex programming

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Abstract. The alternating direction method (ADM) is classical for solving a linearly constrained separable convex programming problem (primal problem), and it is well known that ADM is essentially the application of a concrete form of the proximal point algorithm (PPA) (more precisely, the Douglas-Rachford splitting method) to the corresponding dual problem. In this talk I will show that an efficient method competitive to ADM can be easily derived by applying PPA directly to the primal problem. More specifically, if the proximal parameters are chosen judiciously according to the separable structure of the primal problem, the resulting customized PPA takes a similar decomposition algorithmic framework as that of ADM. The customized PPA and ADM are equally effective to exploit the separable structure of the primal problem, equally efficient in numerical senses and equally easy to implement. Moreover, the customized PPA is ready to be accelerated by an over-relaxation step, yielding a relaxed customized PPA for the primal problem. We verify numerically the competitive efficiency of the customized PPA to ADM, and the effectiveness of the over-relaxation step. Furthermore, I will show a simple proof on the O(1/t) convergence rate of the relaxed customized PPA.