2012 Workshop on Nonlinear Analysis and Optimization

Department of Mathematics
National Taiwan Normal University

November 28-30, 2012

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Organized by

Mau-Hsiang Shih and Jein-Shan Chen

Schedule of Programs
Place : M210, Mathematics Building

Table 1: November 28, Wednesday

	Chair	**Speaker**	Title
09:00 09:40	M-H Shih	W. Takahashi	Nonlinear analytic methods for linear operators in Banach spaces
09:40 10:20	M-H Shih	S. Akashi	Asymptotic behavior of the orbits of the dynamical systems constructed from generalized Collatz-mappings
		Tea Break	
10:40 11:20	H-C Lai	L-J Lin	Bregman distance and related results on Banach spaces
11:20 12:00	H-C Lai	G. M. Lee	On mathematical programs with equilibrium constraints
		Lunch Break	
14:00 14:40	D-S Kim	H-C Lai	Multicriteria minimax programming problem in complex space
14:40 15:20	D-S Kim	M. Hojo	Fixed point theorems and convergence theorems for non-self mappings in Hilbert spaces
		Tea Break	
15:40 16:20	L-J Lin	Y. Gao	Piecewise smooth Lyapunov function for a nonlinear dynamical system
16:20 17:00	L-J Lin	H-K Xu	Gradient-based proximal methods for compressed sensing

Table 2: November 29, Thursday

	Chair	**Speaker**	Title
09:00 09:40	J-S Chen	M. S. Gowda	Z-transformations in complementarity theory and dynamical systems
09:40 10:20	J-S Chen	C. B. Chua	A barrier-based smoothing proximal point algorithm for nonlinear complementarity problems over closed convex cones
		Tea Break	
10:40 11:20	G. M. Lee	R-L Sheu	Tightening a copositive relaxation for standard quadratic optimization problems
11:20 12:00	G. M. Lee	C-H Ko	Optimal grasping manipulation for multifingered robots using semismooth Newton method
		Lunch Break	
14:00 14:40	R-L Sheu	P-W Chen	A perfect match condition for point-set matching problems using the optimal mass transport approach
14:40 15:20	R-L Sheu	J-H Chen	Optimal policies of non-cross-resistant chemotherapy on a cancer model
		Tea Break	
15:40 16:20	S. Akashi	B-S Tam	Maximal exponents of polyhedral cones
16:20 17:00	S. Akashi	D-S Kim	Nonsmooth semi-infinite multiobjective optimization problems

Table 3: November 30, Friday

	Chair	**Speaker**	Title
09:10 09:50	J-S Chen	Y-G Liu	Optimization and applications for some tomography
10:00 10:40	J-S Chen	C-H Huang	Equilibria of abstract economies for Φ-majorized mappings
10:50 11:30	J-S Chen	S-H Wu	Thomas' Conjecture on Finite Distributive Lattices

Asymptotic behavior of the orbits of the dynamical systems constructed from generalized Collatz-mappings

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Abstract. The Collatz conjecture is a conjecture in mathematics named after Lothar Collatz, who first proposed it in 1937, and several approaches to this problem, which are based on various research areas in mathematics such as number theory, probability theory and computation theory, are developed. Actually, fixed point theoretic methods have not appeared yet.

In this talk, we apply fixed point theory to Collatz conjecture, whiich remains to be solved. exactly speaking, we investigate asymptotic behavior of the orbits of the dynamical systems, which can be constructed from generalized Collatz mappings.

Optimal policies of non-cross-resistant chemotherapy on a cancer model

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Abstract. Mathematical models can be applied to study the chemotherapies on tumor cells. Especially, in 1979, Goldie and Coldman proposed the first mathematical model to relate the drug sensitivity of tumors to their mutation rates. They used their model to explain why alternating non-cross-resistant chemotherapy is optimal with simulation approach (jointly with Guaduskas) and later provided a mathematical proof to their earlier simulation work.

However, Goldie and Coldmans analytical work on optimal treatments majorly focuses on the process with symmetrical parameter settings. Little theoretical results on asymmetrical parameter settings are discussed. In this talk, we recast and restate Goldie, Coldman and Guaduskas model as a multi-stage optimization problem. When asymmetrical parameter settings are assumed, conditions under which a treatment policy can be optimal are derived. In addition, by our approach, Goldie and Coldmans work can be treated as a special and henceforth an alternative proof can be obtained. Some numerical examples will also be included in this talk to illustrate our derived conditions and be used for further discussions.

A perfect match condition for point-set matching problems using the optimal mass transport approach

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Abstract. We study the performance of optimal mass transport-based methods applied to point-set matching problems. The present study, which is based on the L_2 mass transport cost, states that perfect matches always occur when the product of the point-set cardinality and the infinity norm of the curl of the non-rigid deformation field does not exceed some constant.

Let $\{x_i\}_{i=1}^n$ and $\{y_i = T(x_i)\}_{i=1}^n$ be two point sets in $\mathbf{R}^d$, where T is a transform with gradient symmetric-skew symmetric decomposition $\nabla T = T_S + T_A$. Then perfect match always occurs if the maximal ratio of the singular values between T_A and T_S is bounded above by C/n, where $C = 2\pi$ in many cases.

This analytic result is justified by a numerical study of matching two sets of pulmonary vascular tree branch points whose displacement is caused by the lung volume changes in the same human subject. The nearly perfect match performance verifies the effectiveness of this mass transport-based approach.

Authors: Pengwen Chen, Ching-Long Lin, and I-Liang Chern

A barrier-based smoothing proximal point algorithm for nonlinear complementarity problems over closed convex cones

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Abstract. We present a new barrier-based method of constructing smoothing approximations for the Euclidean projector onto closed convex cones. These smoothing approximations are used in a smoothing proximal point algorithm to solve monotone nonlinear complementarity problems (NCPs) over a convex cones via the normal map equation. The smoothing approximations allow for the solution of the smoothed normal map equations with Newton's method, and do not require additional analytical properties of the Euclidean projector. The use of proximal terms in the algorithm adds stability to the solution of the smoothed normal map equation, and avoids numerical issues due to ill-conditioning at iterates near the boundary of the cones. We prove a sufficient condition on the barrier used that guarantees the convergence of the algorithm to a solution of the NCP. The sufficient condition is satisfied by all logarithmically homogeneous barriers. Preliminary numerical tests on semidefinite programming problems (SDPs) shows that our algorithm is comparable with the Newton-CG augmented Lagrangian algorithm (SDPNAL) proposed in [X. Y. Zhao, D. Sun, and K.-C.Toh, SIAM J. Optim. 20 (2010), 1737-1765].

Piecewise smooth Lyapunov function for a nonlinear dynamical system

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Abstract. In this talk, we concern with the viability for a control system. The stability and attraction for a nonlinear dynamical system with nonsmooth Lyapunov functions are studied. The previous results on stability and attraction with a max-type Lyapunov function are extended to the case where Lyapunov function is piecewise smooth. A condition, under which stability and attraction are guaranteed with a piecewise smooth Lyapunov function, is proposed. Taking two certain classes of piecewise smooth functions as Lyapunov functions, related conditions for stability and attraction are developed.

Keywords: Nonlinear dynamical system, stability, region of attraction, Lyapnov functions, nonsmooth analysis, piecewise smooth function.

Z-transformations in complementarity theory and dynamical systems

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Abstract. A square real matrix is a Z-matrix if all its off-diagonal entries are nonpositive. A generalization of this to a closed convex cone is a Z-transformation. Examples include Lyapunov and Stein transformations studied in continuous and discrete linear dynamical systems.

A linear complementarity problem is a problem in optimization that includes, e.g., primal-dual linear programs and bimatrix games. Generalizing this problem to cones, we get a cone (linear) complementarity problem, study of which is the complementarity theory. In this talk, we describe connections between complementarity theory and dynamical systems via Z-transformations.

Fixed point theorems and convergence theorems for non-self mappings in Hilbert spaces

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Abstract. Let H be a real Hilbert space and let C be a nonempty subset of H. Kocourek, Takahashi and Yao [5] introduced a broad class of nonlinear mappings in a Hilbert space which covers nonexpansive mappings, nonspreading mappings [7] and hybrid mappings [11]. A mapping $T : C \to H$ is said to be generalized hybrid if there exist $\alpha, \beta \in \mathbb{R}$ such that

$$\alpha \|Tx - Ty\|^2 + (1-\alpha) \|x - Tx\|^2 \leq \beta \|Tx - y\|^2 + (1-\beta) \|x - y\|^2$$

for all $x, y \in C$, where $\mathbb{R}$ is the set of real numbers. We call such a mapping an (α, β)-generalized hybrid mapping. They proved fixed point theorems and nonlinear ergodic theorems of Baillon's type [1] for generalized hybrid mappings; see also Kohsaka and Takahashi [6] and Iemoto and Takahashi [3]. Very recently, Kawasaki and Takahashi [6] introduced a more broad class of nonlinear mappings in a Hilbert space. A mapping $T : C \to H$ is said to be more generalized hybrid if there exist $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta \in \mathbb{R}$ such that

$$\begin{aligned}\alpha \|Tx - Ty\|^2 + \beta \|x - Ty\|^2 + \gamma \|Tx - y\|^2 + \delta \|x - y\|^2 \\ + \varepsilon \|x - Tx\|^2 + \zeta \|y - Ty\|^2 + \eta \|(x - Tx) - (y - Ty)\| \leq 0\end{aligned}$$

for all $x, y \in C$. Such a mapping T is called an $(\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \eta)$-widely more generalized hybrid mapping. In particular, an $(\alpha, \beta, \gamma, \delta, 0, 0, 0)$-widely more generalized hybrid mapping is called an $(\alpha, \beta, \gamma, \delta)$-normal generalized hybrid mapping; see Takahashi, Wong and Yao [12]. An $(\alpha, \beta, \gamma, \delta)$-normal generalized hybrid mapping is a generalized hybrid mapping in the sense of Kocourek, Takahashi and Yao [5] if $\alpha + \beta = -\gamma - \delta = 1$ and $\varepsilon = \zeta = \eta = 0$. A generalized hybrid mapping with a fixed point is quasi-nonexpansive. However, a super hybrid mapping is not quasi-nonexpansive generally even if it has a fixed point.

In this talk, we first prove a fixed point theorem for normal generalized hybrid non-self mappings in a Hilbert space. In the proof, we show that widely more generalized hybrid

mappings are deduced from normal generalized hybrid non-self mappings and then we prove the fixed point theorem by using Kawasaki and Takahashi's fixed point theorem for widely more generalized hybrid mappings [4]. Next, we prove a weak convergence theorem of Mann's type [8] for widely more generalized hybrid non-self mappings in a Hilbert space. For the proof, we use the demi-closedness property for widely more generalized hybrid mappings in a Hilbert space. Finally, using an idea of mean convergence by Shimizu and Takahashi [9] and [10], we prove a mean strong convergence theorem for widely more generalized hybrid mappings in a Hilbert space. This theorem generalizes Hojo and Takahashi's mean convergence theorem [2] for generalized hybrid mappings.

Co-author: Wataru Takahashi.

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Equilibria of abstract economies for Φ-majorized mappings

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Abstract. An H-space is a topological space X, together with a family $\{\Gamma_D\}$ of some nonempty contractible subsets of X indexed by $D \in \langle X \rangle$ such that $\Gamma_D \subset \Gamma_{D'}$ whenever $D \subset D'$. An H-space X is called an $l.c.$-space, if X is an uniform space whose topology is induced by its uniformity $\mathcal{U}$, and there is a base $\mathcal{B}$ consisting of symmetric entourages in $\mathcal{U}$ such that for each $V \in \mathcal{B}$, the set $V(E) := \{y \in X \mid (x, y) \in V \text{ for some } x \in E\}$ is H-convex whenever E is H-convex. In this talk, we first introduce some basic definitions about the generalized games and abstract economies. Next, we establish a general fixed point theorem in $l.c.$-spaces by using a new KKM principle. Finally, we list some recent equilibrium existence theorems about abstract economies.

Keywords. $l.c.$-space, upper semicontinous, Φ_θ-majorized, equilibrium point.

2000 AMS subject classifications. 47H04, 52A99, 54H25, 58E35, 91A10.

Nonsmooth semi-infinite multiobjective optimization problems

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Abstract. We employ some advanced tools of variational analysis and differentiation to establish necessary conditions for (weakly) efficient solutions of a nonsmooth semi-infinite multiobjective optimization problem (SIMOP for brevity). Sufficient conditions for (weakly) efficient solutions of a SIMOP are also provided by means of introducing the concepts of (strictly) generalized convex functions defined in terms of the limiting/Mordukhovich subdifferential of locally Lipschitz functions. In addition, we propose types of Wolfe and Mond-Weir dual problems for SIMOPs, and explore weak and strong duality relations under assumptions of (strictly) generalized convexity. Examples are also designed to analyze and illustrate the obtained results.

Optimal grasping manipulation for multifingered robots using semismooth Newton method

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Abstract. Multifinered robots play an important role in manipulation tasks. They can grasp various shaped objects to perform point-to-point movement. It is important to plan the manipulation path and appropriately control the grasping forces for multifingered robot manipulation. In this paper, we perform the optimal grasping control to find both optimal motion path of the object and minimum grasping forces. The rigid body dynamics of the object and the grasping forces subjected to the second-order cone (SOC) constraints are considered in optimal control problem. The minimum principle is applied to obtain the system equalities and the SOC complementarity problems. The SOC complementarity problems are further recast as the equations with the Fischer-Burmeister (FB) function. Since the FB function is semismooth, the semismooth Newton method with the generalized Jacobian of FB function is used to solve the nonlinear equations. The simulations of optimal grasping manipulation are performed to demonstrate the effectiveness of the proposed approach.

Multicriteria minimax programming problem in complex space

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Abstract. We overview the various types of complex variable functions in the objective of minimax complex programming. It includs : linear, nonlinear, nonfractional to fractional nondifferentiable functions with two complex variable minimax programming problems.

The main tasks are to establish the Necessary and the sufficient optimality conditions by extra assumptions to the necessary conditions.In this work the duality problems is also an important part. In this talk we give some interpretation to duality forms only.

On mathematical programs with equilibrium constraints

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Abstract. Mathematical programs with equilibrium constraints (MPECs), which are formulated as optimization problems with complementarity constraints, have been the subject of intensive research during the last decades. In this talk, we introduce a relaxed version of the MPEC constant positive linear dependence constraint qualification (MPEC-CPLD) for MPECs, which we call MPEC-RCPLD. We show that the MPEC-RCPLD is strictly weaker but easier to check than MPEC-CPLD and is stronger than the MPEC Abadie constraint qualification (thus, it is an MPEC constraint qualification for M-stationarity), and ensures the existence of local MPEC error bounds under certain additional assumptions. We present that under the MPEC-RCPLDs, the sequences of stationary points, which are produced from regularization schemes of Kanzow and Schwartz, and Kadrani, Dussault and Benchakroun for MPECs, converge to M-stationary points of MPECs. Furthermore, we give examples illustrating our main results.

Mathematics Subject Classification (2010). 49K30, 90C30, 90C33, 90C46.

Keywords. mathematical programs with equilibrium constraints, stationary points, constraint qualifications, error bounds, regularization schemes.

Bregman distance and related results on Banach spaces

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Abstract. In this paper, we first study the properties of Bregman distance and an existence and uniqueness theorem of solution for an optimization problem which is related to Bregman distance. From these results, we study fixed point problems for nonlinear mappings, contractive type mappings, Caritsti type mappings, graph contractive type mappings with the Bregman distance on Banach spaces. We also study some properties of Bregman projection. Our results on the properties of Bregman projection improve recent results of Honda and Takahashi. In fact, we combine the techniques of optimization theory and fixed point theory to study these problems in this paper. Our results and techniques are different from any result and technique on fixed point theorems of nonlinear mappings, contractive type mappings, graph contractive mappings. Furthermore, the results in this paper will have many applications on fixed point theory, optimization problems and nonlinear analysis.

Keywords: Bregman distance; Banach limit; fixed point; conjugate function; Gâteaux differentiable.

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Optimization and applications for some tomography

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Abstract. In this talk, we will introduce some optimization problems in application to X-Ray CT, Optical Tomography, Impedance Tomography and discuss corresponding numerical methods, especially Raw-Action Methods like Kaczmarz Method and Hildreth Method.

Tightening a copositive relaxation for standard quadratic optimization problems

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Abstract. In this talk, we focus on the problem of improving the semidefinite programming (SDP) relaxations for the standard quadratic optimization problem (standard QP in short) that concerns with minimizing a quadratic form over a simplex. We first analyze the duality gap between the standard QP and one of its SDP relaxations known as "strengthened Shor's relaxation". To estimate the duality gap, we utilize the duality information of the SDP relaxation to construct a graph G. The estimation can be then reduced to a two-phase problem of enumerating first all the minimal vertex covers of G and solving next a family of second-order cone programming problems. When there is a nonzero duality gap, this duality gap estimation can lead to a strictly tighter lower bound than the strengthened Shor's SDP bound. With the duality gap estimation improving scheme, we develop further a heuristic algorithm for obtaining a good approximate solution for standard QP.

Nonlinear analytic methods for linear operators in Banach spaces

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Abstract. Recently, two retractions (projections) which are different from the metric projection and the sunny nonexpansive retraction in a Banach space were found. In this talk, using nonlinear analytic methods and new retractions, we prove some theorems which are related to linear operators in Banach spaces.

Maximal exponents of polyhedral cones

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Abstract. Let K be a proper (i.e., closed, pointed, full convex) cone in $\mathbb{R}^n$. An $n \times n$ matrix A is said to be K-primitive if $AK \subseteq K$ and there exists a positive integer k such that $A^k(K \setminus \{0\}) \subseteq \operatorname{int} K$; the least such k is referred to as the exponent of A and is denoted by $\gamma(A)$. For a polyhedral cone K, the maximum value of $\gamma(A)$, taken over all K-primitive matrices A, is denoted by $\gamma(K)$. It is proved that for any positive integers $m, n, 3 \leq n \leq m$, the maximum value of $\gamma(K)$, as K runs through all n-dimensional polyhedral cones with m extreme rays, equals $(n-1)(m-1) + \frac{1}{2}\left(1 + (-1)^{(n-1)m}\right)$. For the 3-dimensional case, the cones K and the corresponding K-primitive matrices A such that $\gamma(K)$ and $\gamma(A)$ attain the maximum value are identified up to respectively linear isomorphism and cone-equivalence modulo positive scalar multiplication.

This is a joint work with Raphael Loewy and Micha A. Perles.

Thomas' Conjecture on Finite Distributive Lattices

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Abstract. In 1981, the biologist René Thomas conjectured that the presence of a negative circuit in the interaction graph of a dynamical system is a necessary condition for this system to produce sustained oscillations. Shih and Dong stated and proved the Jacobian conjecture for boolean algebra. Shih-Dong's theorem provides a framework for Thomas' conjecture. We now present Shih-Dong's theorem to finite distributive lattices.

This is a joint work with professor Juei-Ling Ho.

Gradient-based proximal methods for compressed sensing

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Abstract. Compressed sensing (CS) is a novel sensing/sampling technique which asserts that one can recover certain signals and images from far fewer samples or measurements than traditional methods use. The cornerstone of the CS theory consists of two principles: sparsity and incoherence. Sparsity pertains to the signals of interest and incoherence to the sensing modality. Mathematically, one of CS amounts to recovering a possibly sparsest solution x from an underdetermined linear system $Ax = b$, where A is an $m \times n$ matrix such that $m \ll n$.

In this talk we will discuss how gradient-based proximal methods can be applied to iteratively recover the signal x after convexly relaxed as an ℓ_1 regularization problem.