

Exercise (Week 6)

October 13, 2022

1. You have a five points in the plane, not all colinear. Prove that there is a line passing through just two of these points. (平面上有不共線的 5 個點。證明存在一條線僅通過其中兩個點)
2. Given $a, b \in \mathbb{N}$ with $b > 1$, prove that if there exist $h, r \in \mathbb{Z}$ with $0 \leq r < b$ such that $a = bh + r$, then h is unique and r is unique.
3. Use the pigeonhole principle to prove that no matter how 7 points are placed within an 8 by 9 rectangle, there will always be a pair whose distance is at most 5. (在 8×9 的長方形中任點 7 個點，一定有兩個點其距離不大於 5) (Hint: $3^2 + 4^2 = 5^2$)
4. Prove the pigeonhole principle by mathematical induction.
5. Using (strong) mathematical induction to prove the following:
 - (a) If $a_1 = 1$ and $a_n = 2a_{n-1} + 1$, for $n \geq 2$, then $a_n = 2^n - 1$ for all $n \in \mathbb{N}$.
 - (b) If $b_1 = 1$ and $b_n = n + \sum_{i=1}^{n-1} b_i$, for $n \geq 2$, then $b_n = 2^n - 1$ for all $n \in \mathbb{N}$.
6. Do Question 2.9.