

Exercise (Week 7)

October 19, 2022

1. Let

$$A = \{n \in \mathbb{N} \mid n^2 + 5\}, \quad B = \{n^2 + 5 \mid n \in \mathbb{N}\}, \\ C = \{n \in \mathbb{N} \mid n^2 + 5 > 35\}, \quad D = \{n^2 + 5 > 35 \mid n \in \mathbb{N}\}.$$

- (a) Two of the expressions above describe sets. Find Them. (以上哪兩個是正確集合表示?)
- (b) Write 3 elements for each of the two sets.
- (c) Is one of the set a subset of the other? Show your reasoning.

2. Let

$$S = \{n \in \mathbb{N} \mid 1 \leq n \leq 5\}, \quad T = \{n \in \mathbb{N} \mid 5 \leq 2n - 1 \leq 17\}, \\ U = \{2n - 1 \mid n \in S\}, \quad V = \{2n \mid n = 1, 2, 3, 4\}.$$

Calculate each of the following. (Hint: List all the elements of the set)

- (a) $S \cap U$.
- (b) $(S \cap T) \cup U$.
- (c) $S \cap (T \cup U)$.
- (d) $(S \cup T) \cap V$.

3. Let $S = \{x \in \mathbb{R} \mid 2 < x < 9\}$, $T = \{y \in \mathbb{R} \mid 5 \leq y < 14\}$.

- (a) Prove $S \cap T = \{z \in \mathbb{R} \mid 5 \leq z < 9\}$.
- (b) Write $S \cup T$ and prove your answer.

4. Let A, B, C, D be sets.

- (a) Prove that if $A \subseteq B$ and $C \subseteq B$, then $A \cup C \subseteq B$, by showing that every element of $A \cup C$ is an element of B .
- (b) Using B and D are both a subset of $B \cup D$ and (a) to prove that if $A \subseteq B$ and $C \subseteq D$, then $(A \cup C) \subseteq (B \cup D)$.

5. Let C, D be sets.

- (a) Which one of the following statements is equivalent to $C \not\subseteq D$.
 - i. If $x \in C$, then $x \notin D$.
 - ii. $x \in C$ and $x \notin D$.
 - iii. If there is an $x \in C$, then $x \notin D$.
 - iv. There is an $x \in C$ such that $x \notin D$.
- (b) Using your answer for (a) to explain why $\emptyset \not\subseteq D$ is wrong.

6. Do Question 3.4.