Exercise (Week 9)

November 03, 2022

1. Let $I = \{i \in \mathbb{N} : i \geq 2\}$ and $\forall i \in I$, let $A_i = \{m/i : m \in \mathbb{Z}\}$.

   (a) Prove for any $k \in I$, $\bigcup_{i=k}^{\infty} A_i = \mathbb{Q}$.

   (b) Is it possible to find $j, k \in I$ with $j < k$ such that $\bigcap_{i=j}^{k} A_i = \mathbb{Q}$?

2. Let I be the closed interval $[1,2] = \{r \in \mathbb{R} : 1 \leq r \leq 2\}$. Suppose that for every $r \in I$, A_r is a set and $A_r \subseteq A_s$ for all $r, s \in I$ with $r > s$. We know that there exist $i, j \in I$ such that

$$\bigcap_{r \in I} A_r = A_i, \quad \bigcup_{r \in I} A_r = A_j.$$

   Find i and j. (Please show your reasoning.)

3. Let A_i, B_i are sets with index set I. Find an example to show the following are not always true.

$$(\bigcap_{i \in I} A_i) \cup (\bigcap_{i \in I} B_i) = \bigcap_{i \in I} (A_i \cup B_i),$$

$$(\bigcup_{i \in I} A_i) \cap (\bigcup_{i \in I} B_i) = \bigcup_{i \in I} (A_i \cap B_i).$$

   Please find correct statements for them with proof.

4. Do Questions 3.13 & 3.18