Exercise (Week 10)
November 10, 2022

1. Let A, B, C, D be sets.

   (a) Explain why $(C \setminus A) \times (D \setminus B) = ((C \setminus A) \cap C) \times (D \cap (D \setminus B))$.

   (b) Use (a) to prove $(C \setminus A) \times (D \setminus B) = ((C \setminus A) \times D) \cap (C \times (D \setminus B))$.

   (c) Use (b) to prove $(C \setminus A) \times (D \setminus B) = (C \times D) \setminus ((A \times D) \cup (C \times B))$.

   (d) Explain why $(C \times D) \setminus (A \times B) = (C \times D) \setminus ((C \times D) \cap (A \times B))$.

   (e) Use (d) to prove $(C \times D) \setminus (A \times B) = (C \times D) \setminus ((C \times B) \cap (A \times D))$.

   (f) Use (e) to prove $(C \times D) \setminus (A \times B) = (C \times (D \setminus B)) \cup ((C \setminus A) \times D)$.

2. For a given set A, let $\mathfrak{T}_A = \{(a, S) \in A \times \mathscr{P}(A) : a \in S\}$.

   (a) Please describe $\mathfrak{T}_\emptyset$, $\mathfrak{T}_{\{a\}}$ and $\mathfrak{T}_{\{a,b\}}$ using list mehthod.

   (b) Let A be a finite set of n elements, find the number of elements of $\mathfrak{T}_A$. (you can use the fact that $\#(\mathscr{P}(A)) = 2^n$.)

   (c) Let A, B be sets. Is it true that $\mathfrak{T}_{A \cap B} = \mathfrak{T}_A \cap \mathfrak{T}_B$? Explain your answer.

3. Define a relation on $\mathbb{R}$ by setting $x \sim y$ if and only if $|x - y| < 1$. Is this relation an equivalence relation? Explain your answer.

4. Let $X = \{1, 2, 3\}$ and suppose $S \subseteq X \times X$ is an equivalence relation on X. Suppose further that $(1, 2) \in S$ and $(2, 3) \notin S$. Find all the elements of S.

5. Do Question 4.3.