

Exercise (Week 11)

November 17, 2022

1. Let $X = \{1, 2, 3\}$. How many different equivalence relations can we find on X (在 X 中可以定義多少種 equivalence relation)?
(Hint: X 的 partition 和 X 的 equivalence relation 有著一對一的對應關係.)
2. Let (\mathbb{C}, \prec) be a strict total ordered set with the following additional properties:
A: If $a \prec b$, then for every $c \in \mathbb{C}$, $a + c \prec b + c$
M: If $a \prec b$ and $0 \prec c$, then $ac \prec bc$
 - (a) Prove if $a \prec 0$, then $0 \prec -a$.
 - (b) Prove $0 \prec c^2$, $\forall c \in \mathbb{C}$.
3. Let A, B be nonempty subsets of a partial order set (X, \preceq) . Suppose that $A \subseteq B$.
 - (a) Prove that if there exists an upper bound of B , then there exists an upper bound of A .
 - (b) Suppose that a and b are the least upper bound of A and B , respectively. Prove that $a \preceq b$.