

Exercise (Week 13)

December 01, 2022

1. Consider the function $f: \{1, 2, 3\} \rightarrow \{1, 2, 3\}$ defined by $f(1) = f(3) = 2$ and $f(2) = 3$. List all subsets $A \subseteq \{1, 2, 3\}$ and find $f(A)$ and $f^{-1}(A)$.
2. Suppose that $f: X \rightarrow X$ is a function and $A \subseteq X$. Which of the following statements are true? Explain each case.

$$\begin{array}{ll} (a) f(A^c) \subseteq f(A)^c & (b) f(A)^c \subseteq f(A^c) \\ (c) f^{-1}(A^c) \subseteq f^{-1}(A)^c & (d) f^{-1}(A)^c \subseteq f^{-1}(A^c) \end{array}$$

3. Suppose that $g: X \rightarrow Y$ is a surjective function and $B \subseteq Y$.
 - (a) Prove that there exists $A \subseteq X$ such that $g(A) = B$.
 - (b) Prove $g(g^{-1}(B)) = B$.
 - (c) Suppose $B' \subseteq Y$ and $B' \neq B$. Prove $g^{-1}(B') \neq g^{-1}(B)$.
4. Find functions $f: \mathbb{N} \rightarrow \mathbb{N}$ and $g: \mathbb{N} \rightarrow \mathbb{N}$ such that f is not onto and yet $g \circ f$ is onto.