Correction to *Sobolev functions on varifolds*

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December 9, 2025

Abstract

This note corrects the proof of Theorem 3.6 of *Sobolev functions on varifolds* in *Proc. Lond. Math. Soc. (3)*, 113(6):725–774, 2016.

Assuming the typographical error in [Men16, Lemma 3.4] to be removed ($\mathbf{R}^n$ on its third line should be U), it cannot be applied in the proof of [Men16, Theorem 3.6] to construct $g : U \to \mathbf{R}^l$ as the domain of the resulting function could fail to contain U. As a remedy, we provide an extended version which reduces to the typographically corrected version of the original when $H = U$.

Lemma 3.4 (extended version)**.** *Suppose $l, n \in \mathscr{P}$, U and H are open subsets of $\mathbf{R}^n$, $A \subset H \subset U$, the closure of A relative to U is contained in H, $f : H \to \mathbf{R}^l$ is of class 1, and $\varepsilon > 0$.*

Then, there exist an open subset X of H and a function $g : U \to \mathbf{R}^l$ of class 1 such that $A \subset X$, $f|X = g|X$, and

$$\operatorname{Lip} g \le \varepsilon + \sup\{\operatorname{Lip}(f|A), \sup \|\operatorname{D} f\|[A]\}.$$

Moreover, if $l = 1$ and $f \ge 0$ then one may require $g \ge 0$.

Proof. We replace U by H in lines 2 and 5 of the original proof. □

Theorem 3.6 (unchanged)**.** *Suppose $l, m, n \in \mathscr{P}$, $m \le n$, U is an open subset of $\mathbf{R}^n$, $V \in \mathbf{RV}_m(U)$, C is a relatively closed subset of U, $f : U \to \mathbf{R}^l$ is locally Lipschitzian, $\operatorname{spt} f \subset \operatorname{Int} C$, and $\varepsilon > 0$.*

Then there exists $g : U \to \mathbf{R}^l$ of class 1 satisfying

$$\operatorname{spt} g \subset C, \quad \operatorname{Lip} g \le \varepsilon + \operatorname{Lip} f, \quad \|V\|(U \sim \{x : f(x) = g(x)\}) \le \varepsilon.$$

Moreover, if $l = 1$ and $f \ge 0$ then one may require $g \ge 0$.

Proof. We replace the first sentence of the original incorrect proof by "We firstly construct a relatively closed subset D of U such that

$$\operatorname{spt} f \subset \operatorname{Int} D, \quad D \subset \operatorname{Int} C, \quad \|V\|(\operatorname{Bdry} D) = 0,$$

and let $X = \operatorname{Int} D$." and its last sentence by "Choosing an open set W such that

$$(X \sim B) \cup \operatorname{Bdry} D \subset W \subset C \quad \text{and} \quad \|V\|(W) \le \varepsilon$$

and recalling $B \subset M \subset G \subset X$, we let $H = G \cup (U \sim D)$ and $A = U \sim W \subset H$, note

$$A \cap G \subset B, \quad (h \circ r)|(A \cap G) = f|(A \cap G), \quad f|(U \sim D) = 0,$$

and apply Lemma 3.4 (extended version) with f replaced by $(h \circ r) \cup (f | (U \sim D))$ to obtain a function $g : U \to \mathbf{R}^l$ of class 1 such that

$$g | A = f | A, \quad \operatorname{Lip} g \leq \varepsilon + \operatorname{Lip} f,$$

and $g \geq 0$ if $l = 1$ and $f \geq 0$; these conditions entail the conclusion because $U \sim C \subset A \sim D$ and $U \sim A \subset W$." □

We are grateful to M. Workman for pointing out the errors corrected above.

Reference

[Men16] Ulrich Menne. Sobolev functions on varifolds. *Proc. Lond. Math. Soc. (3)*, 113(6):725–774, 2016. URL: `https://doi.org/10.1112/plms/pdw023`.

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